“TIME IS WHEN YOU WAIT:” AN EXPLORATION OF CHILDREN’S CONCEPTIONS OF DURATION AS AN ATTRIBUTE OF THEIR LIVED EXPERIENCES

by

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"Time is When You Wait:" An Exploration of Children's Conceptions of Duration as an Attribute of Their Lived Experiences

Dissertation directed by Professor Ron Tzur

ABSTRACT

In response to Earnest (2019) who asserted "too often studies involving time in maths education are divorced from duration, the quantity that time measures…therefore future studies ought to incorporate the experience of duration into the study design" (p. 25), the purpose of this study was to begin mapping possible conceptualizations children have of duration as an attribute of their lived experiences prior to formal, school-based instruction on time. Specifically, the following research question was posed:

> When reflecting on past experiences, what conceptions do children (grades preK-1, ages 4-6 years old) seem to have about the duration of those experiences, as indicated by their descriptions of duration?

Broadening the current methodology for investigating children's durational reasoning, I drew from phenomenology (Creswell, 2013; Hycner, 1999) and clinical interview case studies (Clement, 2000) to explore seven children's descriptions of the duration of their lived experiences. I designed semi-structured interviews questions to promote the participants' re-presentation of past experiences, or mental operations of reconstructing past experiences (von Glasersfeld, 1991) to conceive of duration as an attribute of those experiences, rather than reflecting on in-the-moment durational experiments.

Utilizing Wolcott's (1994) *Description*, *Analysis*, and *Interpretation*, I distinguished three themes from the participants' descriptions: (a) duration as an accumulation of activities

completed; (b) duration as a consideration of a gross quantity; and (c) duration as a result of exertion. From my interpretation of these themes, I postulate a possible development of durational units (von Glasersfeld, 1981) and a correspondence between conception of number (Piaget, 1965) and duration. Additionally, my findings seem to support Piaget's (1969) conclusions of children's development of inner duration through their perceived rapidity and efforts. I discuss implications of my findings for educators, including a possible re-alignment of Common Core Standards (CCSSI: Measurement and Data, 2020) for school-based instruction on time. I conclude with two methodological limitations of this dissertation and suggest future research based on these limitations.

The form and content of this abstract are approved. I recommend its publication.

Approved: Ron Tzur

DEDICATION

For my dad—who asked a wide-eyed 11-year-old when forever ends.

I wish you had been able to see this.

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INTRODUCTION

> …[W]hen retrospective [timing] judgements are unexpectedly called for, it is likely that [the individual has] not carried out any deliberate timing, and so the duration judgements must come from something else. But what exactly is this "something else"? (Wearden, 2016, p. 119)

The purpose of this study was to begin mapping what conceptualizations children might have of duration as an attribute of their experiences prior to formal school-based instruction on time. Specifically, I intend this study to explore what conceptual markers, or as Tzur (2019) described, "available ways of reasoning mathematically" (p. 60) 4-, 5-, and 6-year-olds might have about the duration of their past experiences. This exploration was in response to Earnest (2019), who stated:

> [T]oo often studies involving time in maths education are divorced from duration, the quantity that time measures…Research on children's time-related ideas ought to consider not just symbol systems for time but how such symbol systems speak to the duration as an ongoing experience, and therefore future studies ought to incorporate the experience of duration into the study design. (p. 25)

Exploring these experiences is important for several reasons. First, the present body of research seems to be lacking markers of such conceptions, as noted by Earnest (2019)—specifically, indicators of how children re-present (or mentally recall and re-run) the time of their lived experiences, what Piaget (1969) referred to as *psychological time*. Additionally, how children come to reason about duration as the measurable quantity of time can impact their

reasoning about time as a variable in higher-level math and science. For example, how time might be understood as an independent variable in covariational reasoning (Ellis et al., 2015; Lobato, et al., 2012; Thompson, 2012) and the scientific use of time in physics (Brookes, 2006) through concepts such as motion (Singh, 2016) and acceleration (Tasar, 2010).

In this study, I will focus on what conceptions preschool, kindergarten, and first-grade children (ages 4, 5, and 6 years old, respectively) might have as they re-present the duration of familiar experiences. I conjecture that these conceptions are based on the children's intuitive observations of the world (Piaget, 1969) and accepted ideas from peers, parents, and other people in their social milieu (Lareau, 2011; Levine, 1997; Piaget, 1969).

In this chapter, I will begin by grounding my study within the current scope of elementary teaching and learning. I will then turn to how time has been characterized by Western theorists, such as Einstein (1920) and Piaget (1969). Finally, to situate my conceptual framework, I will provide an example of how one child reasoned about duration. But first, I present my own experience of the durational reasoning of adults to demonstrate how such durational conceptions can pervade our everyday lives.

An Example of Durational Reasoning in Everyday Life: The Baby Shower

I recently attended a baby shower and witnessed how challenging reasoning about duration can be. Baby showers vary in experience for those in attendance. For the happy mom-to-be, the shower "flies by" in well-wishes and baby-themed games. But for many guests, a three-hour baby shower may seem to "drag on" as they feel pressed to participate in prearranged merriment.

This baby shower began at 12:00pm (noon). We were all standing around socializing, when a guest standing near me glanced over at a clock hanging on the wall (see Figure 1). She

exclaimed, “Oh my gosh, it’s already 2:00? I have to go!” I looked at the clock and I saw it showed the hour of 12 and 10 minutes.

Figure 1

Analog clock showing 12:10 (Clipart ETC, 2009)

It is true that the alignment of the hands were in a similar (but not the same) position to how they would appear at 2:00. Additionally, the wall on which the clock was hanging was at an angle from where we were standing, making the two hands appear more or less the same length. What struck me most, however, was the fact that this guest had been standing around talking for 10 minutes, yet, did not seem to reason about the durational difference between 10 minutes and 2 hours.

This (mis)perception[1] was about more than clock reading or elapsed time calculation. In fact, I assume this guest could calculate the 10-minute elapsed time. Rather, this (mis)perception was about how an adult conceptualized her experience of duration and used that experience to quantify time. This woman’s experience and conceptualization seems relevant for understanding how people in the United States conceptualize duration. I consider this issue to reflect the lack of explicit teaching how to reason about duration as a quantity in U.S. schools, to which I turn in the following section.

[1] I use the term (mis)perception because, following a constructivist framework, the woman had a perception that was different than my own. I wish to highlight that from other frames of reference, the woman’s perception seemed “mis,” however, for her, it made perfect sense.

Background: Teaching and Learning of Time in the United States

What time is, and how one may come to understand and use this concept, are questions that seem to align with their learning of early elementary mathematics. Children in the United States learn time concepts in two very different ways. One way they learn is in the classroom, through teaching that focuses on analog and digital clock reading and elapsed time calculations (Boulton-Lewis et.al, 1997; Burney et al., 2009, 2012; CCSSI: Measurement and Data, 2020; Freidman & Laycock, 1989; Harris, 2008; Kamii & Russell, 2012; Korvorst et al., 2007; Meeuwissen et.al, 2004; Reisman, 1971). A second way they learn is through their daily experiences and interactions with others (Lareau, 2011; Levine, 1997). Typically, school-based instruction on time seems to take for granted these temporal experiences. Children are instead taught procedures to read clocks (e.g., when the long hand is at the 12 you say "o'clock"), and how to make arbitrary elapsed time calculations (e.g., Spencer left the house at 7:15 and got to school at 7:35, how long did it take Spencer to get to school?). These skills are isolated from the durations they describe, let alone from a child's experience of such durations. They are, by-in-large, based in procedures, not conceptions, and can therefore lead to perceptions about time, such as the woman's misreading of the clock at the baby shower.

Currently, formal, school-based instruction on time begins at first grade, when children are around 6 years old. According to the Common Core State Standards (CCSSI: Measurement and Data, 2020), first grade students will "Tell and write time in hours and half-hours using analog and digital clocks." In my scrutiny of the CCSSI, I found no prerequisite standards that focus on children's learning *what it is* that analog and digital clocks measure, what hours and minutes are, let alone what durational time itself is. Thus, the CCSSI seem based on either an assumption that children have these understandings before they enter first grade (roughly by the

age of 6), or that they are not necessary for time telling and, later, elapsed time calculations. Alternatively, CCSSI might simply overlook this issue. One possibility for overlooking children's experience and conceptualization of duration could be that it is unknown how children come to conceptualize time; thus, it is taken for granted. Considering the guest at the baby shower, however, the implications for how time is conceptualized seem to extend well beyond the elementary classroom.

This lack of conceptual prerequisites seems counter to how CCSSI considers other forms of measure, such as linear measurement. According to CCSSI (Measurement and Data, 2020), kindergarteners should be able to "Describe measurable attributes of objects, such as length or weight" and "Directly compare two objects with a measurable attribute in common" prior to learning standard or non-standard tools of measurement. This seems paradoxical; kindergarteners are supposed to be explicitly taught about the attributes they are measuring before being taught how to use the tools to measure them (Tzur, 2008). For example, a kindergartener might be taught in school to compare two lengths of string to tell which is longer (or shorter). This comparison is intended to later bring about strategies for measuring, such as aligning starting points or considering leftovers, both important considerations when accounting for time (D. Earnest, personal conversation, May 27, 2019). Time, however, does not seem an attribute in the same, root-concept sense. It is not tangible and cannot be directly compared against itself; however, one can conceptualize time through their experiences, as I will explicate in Chapter II. Time is the measurement of an "invisible quantity" (Earnest, 2019). I will now elaborate on my interest in how children might come to recognize such a quantity, beginning with how time has been acknowledged by others.

Theoretical Basis for the Study

To situate how children might come to conceptualize the invisible quantity we call duration, I will narrow the extensive field of time literature, beginning with an epistemological review of how the concept of time has been explained by Western theorists of the past. Based on this brief chronological review, I will explicate the theoretical lens for this study, specifically positing durational reasoning as a cognitive construction, as asserted by Piaget (1969), among others (Kamii & Russell, 2010, 2012; Long & Kamii, 2001; Russell, 2008). Building from this constructivist framework, I will characterize *time* as a measurable attribute of experience using von Glasersfeld's (1981) attentional model, von Glasersfeld and Steffe's (1985; Steffe, 1991) unitizing operation, and Long and Kamii's (2001) criteria for the measurement of time.

Epistemological Review of the Concept of Time

Philosophers, psychologists, scientists, and mathematicians from around the world have been studying and characterizing time for millennia. These characterizations have not produced a common definition for what time is, nor how individuals understand it.

In 300 BCE, Aristotle theorized that time was a kind of number—specifically, the number of change (movement) in one's experiences of before and after (2012, Book 3, Part 6-7). According to Aristotle, time is not naturally finite and countable, the way that pebbles or cubes are countable (e.g., one, two, three). Instead, time is continuous, and only becomes countable when potentially divided into segments (Coope, 2005). This potential division occurs through experience of some change. When a change in activity or attention occurs, time might be mentally (and, later theoretically) partitioned, thus allowing for the recognition of now versus previous "nows," as termed by Aristotle (Coope, 2005, p. 90). Aristotle asserted, "time's relation

to a now is like a number's relation to a unit" (Coope, 2005, p. 86), and is therefore constructed within the mind of the individual.

Building on Aristotle's definition, Descartes (1644/2017) defined time as the measure of motion. He added that time became a mode of thought "in the same way number, when it is not considered as in created things, but merely in the abstract or in general, is only a mode of thinking" (Descartes, 2017, p. 29, Part 1, LVIII) when distinguished from duration. He called this a number of 'movement.' The assertion that an individual creates time in the mind was further described by Hume (1737/2001) and Kant (1781/2007), who both explained time as a mental representation.

Hume described time as particular successions of perceptions, where an abstract representation of quantity and quality was created (2001, Part 2, Section 3). He argued that "time, being deriv'd from the succession of our perceptions of every kind, ideas as well as impressions, and impressions of reflection as well as of sensation, will afford us an instance of an abstract idea" (Hume, 2001, p. 28, SB35). Thus, to Hume, time is inextricably linked to succession and it "is just as impossible to conceptualize time without successive perceptions as it is to conceptualize a line without any particular length" (Esposito, 2010, p. 15).

Hume assumed that time is understood through perceived experiences; Kant viewed time as "prior to experience, not by means of experience" (Kant, 2007, p. B46,47|A31,32). According to Kant, time, along with space, was an a priori representation that underlies all intuition; it was a universal condition (2007, Transcendental Doctrine of Elements, Section II). That is, to Kant the conceptualization of time cannot develop through experience because time is a necessary condition for experience.

Many theorists argued against Kant's position of a priori time, including Bergson (1910/2015) and Piaget (1969), like Hume, believed that the individual constructed time within the mind and understood it through their experiences. According to von Glasersfeld (1981), to Piaget, time, along with space and number, is "the [construction] of an active mind that conceptually organizes both itself and its world by crystallizing experience into an interaction of experiencing subject and the objects of experience" (p. 83). Bergson (1910/2015, 1912), however, posited that an individual could not represent time by images or concepts; it only existed through "qualitative multiplicity"—uncountable, progressive subjective experience. An individual could not measure this "pure duration" (Bergson, 1912, p. 13) experience without losing the "richness of color, characteristic of duration that is lived" (p. 15). When an individual measured time, Bergson argued, it was being confused with space.

Bergson's distinction between time and space was not consistent with Einstein's theory of space-time (Canales, 2016). In his Special (1905) and General (1906) *Theory of Relativity*, Einstein posited that experiences exist in a continuum of space and time. Because this continuum is always there, how an individual might perceive these experiences is relative. Specifically, the time of experience is depended on the individual's position in space. Recalling the woman from the baby shower, her position relative to the clock might have contributed to her misreading of the time as 2 o'clock instead of 12:10. This led her to believe that 2 hours had passed, a duration 12 times as long as the 10 minutes that had actually elapsed. Her physical location and the experience of (incorrectly) reading the clock hands determined her experienced time. Likely, her experience did not involve an awareness of the space aspect, to which I pointed out as an observer.

To address differences among people's experiences of simultaneity, in *The Special Theory of Relativity* (1905), Einstein described a train moving along a track. While traveling, two bolts of lightning strike two points at the same moment—'A' striking behind the train, 'B' striking in front of the train. For an onlooker standing next to the track at the exact midpoint between the two strikes, the events happen simultaneously. Strike 'A' and 'B' hit (the onlooker's eyes) at the exact same moment in time. However, for a passenger on the train moving toward strike 'B' and away from 'A,' the light emitted from 'B' precedes the light from 'A.' Being closer to strike 'B' means that the speed at which that light reached the passenger was shorter than strike 'A.' This difference between observers' experiences, Einstein argued, is relativity. For my focus on children's conceptualization of their experience of duration, it further emphasizes the idiosyncrasy of one's experience.

Reflecting, again, on the guest at the baby shower, her position compared to my own position in the room, and most likely also our individual experiences of the event, caused two different time readings from the same clock. Because I was able to interpret the clock accurately as 12:10, hearing it being read by someone else as 2 o'clock caused me to experience a discrepancy with my own time reasoning. I had to coordinate my own experience of the 10 minutes aligning with the 12:10 that I saw on the clock, with hearing the guest say 2 hours had elapsed, which illuminated another person's experience. My conceptualization of what 10 minutes feels like, and the types and amount of activities that I can accomplish in that duration, aligned with my experience (at least) in that moment. When the guest said, "2 o'clock," my experience included a gut reaction that it was not possible I had been at the baby shower for 2 hours. This brought forth my reflection on my own perception of the time compared to hers.

In a more "everyday" scenario, Einstein posited, "When you sit with a nice [person] for two hours, it seems like two minutes; when you sit on a hot stove for two minutes it feels like two hours. That's relativity" (Einstein, as cited by Levine, 1997, p. 26). Here, again, we see that the perception of one's experiences influences durational reasoning. However, rather than perception being situated in a specific location in space, like the observer's position relative to the lightning in the previous train example, a specific experience situated the perception, like the over-looker vs. the passenger, or the guest at the baby shower. The guest conceptualized the duration of her experience based on her perception of the time she read. She did not question the time she read on the clock but rather, she questioned her experience of the duration. In other words, she did not suspect that she misread the clock or perhaps that the clock was broken, but rather, she seemed surprised by her experience of how "time had flown by" during her friendly conversation. In both the train scenario and the baby shower, time was relative to the individual and caused different durational judgements. Such individual differences in one's experiences would seem central to how an individual child may conceptualize this experience.

Piaget (1969) similarly defined time through space. According to Piaget, time, along with space, could not be "perceived [nor] conceived apart from the entities or the events that fill it" (p. 1). Thus, to Piaget, time is an attribute of human experience. Understanding how an individual, specifically a child, perceives and conceives time can be challenging, as many early time conceptions are constructed based on how adults use and discuss time, not necessarily the child's own temporal experiences (Piaget 1969). How children develop their own conceptions of time was the focus in his seminal text, *The Child's Conception of Time* (Piaget, 1969), to which I now turn.

Piaget's The Child's Conception of Time (1927/1969)

In his seminal study of *The Child's Conception of Time,* Piaget (1969) investigated how elementary-age children explained physical and psychological time through spatial experiences, such as vessels filling with liquid or walking versus running to school. Piaget (1969) asserted that young children develop an intuitive sense of time based on their perceptions of succession (the sequence in which events occur) and duration (the experienced intervals that lapse between said events, i.e., start and end of some given experience). Children gain these perceptions through their interpretation of the time relationships they experience. For example, young children equate age (duration) with height (length), believing that people who are taller are inherently older (Piaget, 1969). The child experiences this relationship first-hand as they both age and grow simultaneously, as well as through interactions with others, such as parents and siblings. From experiences such as these, the resulting *conception*, or "way of knowing" as characterized by Simon and colleagues (2004, p. 306), is the child's coordination of the attributes "older" and "taller" into a single construct. To a child reasoning intuitively about time, initially there is no distinction between older and taller; they are indeed one and the same (Piaget, 1969).

Through reflection on experiences, children's conceptions of time develop, shifting from intuitive to operational (Piaget, 1969), that is, to a form of logico-mathematical knowledge. Logico-mathematical knowledge develops from the relationships constructed from observations of actions, for example, a child reasoning that running will get them to a place faster than walking. When a child can reason about the inverse relationship between duration and activity (i.e., faster action creates shorter durations and slower action creates longer durations), they are reasoning operationally (logically). Most importantly, reasoning about time operationally is necessary for durational reasoning, and ultimately for the measurement of time (Kamii &

Housman, 2000; Kamii & Russell, 2010, 2012; Long & Kamii, 2001; Piaget, 1969; Russell, 2008).

Building from Piaget's (1969) *The Child's Conception of Time* and Piaget and colleagues (1960) *The Child's Conception of Geometry*, Long and Kamii (2001) stated, "The purpose of measurement is to make indirect comparisons (relationships)" (p. 125). Two types of reasoning are needed for measurement: transitive reasoning and unit iteration (Long & Kamii, 2001; Piaget, 1969; Piaget et al., 1960). Transitive reasoning involves comparing one whole against another. In durational reasoning, these wholes might be single events, for example comparing the lengths of two songs (Long & Kamii, 2001). Unit iteration is the understanding that a whole can be composed of equal, repeatable parts. For example, reasoning about how many episodes of a TV show you could watch in relation to how long it might take to get to grandma's house (Earnest, 2018a). The same mental operation is necessary for considering standardized durational units of seconds creating minutes creating hours (Long & Kamii, 2001).

Standard units of time, much like inches found on a ruler, are the result of past operations (Piaget, et.al, 1960). Accordingly:

> There is no point in studying how children use ready-made foot rules until we have investigated the way they make up their own foot rules or mensural units, even if the latter are crude to a degree and serve only a momentary purpose. (p. 27)

And, while durational reasoning differs from other length-based measurements in its intangible nature, how children create their own durational units seems important to understand how they might operate with standard tools of time. The following is an example of how one child, 4-year-old Matti (pseudonym), reflected on the duration of his experiences.

An Example of a Child's Conception of Time

To illustrate how a child might conceptualize time through experience, I present a conversation with a preschooler (age 4) about his durational measurement of a common, daily experience of brushing his teeth, shown in Excerpt 1.1.

Excerpt 1.1 (March 2019): Matti's Description of the Duration of Brushing his Teeth

1. A (Author): How long does it take you to brush your teeth?
2. M (Matti): My mom says the ABCs and at the end of it I spit out my toothpaste.
3. A: Can you show me how your mom says the ABCs?
4. M: Mmhmm, so [begins singing to the tune of the ABC song] ABCD, ABC, HIKA, LMN ooh T, QRS, TUV, WS, 9 and Z. Now I know my ABCs, HI know my ABCs.
5. A: And then you spit after all that?
6. M: Mmhmm [nodding][2]

During the conversation, Matti re-presented to himself the duration of a specific experience and described his experience to me without the use of standard durational language (i.e., minute, time, moment, etc.). Instead, he conceptualized the time of brushing his teeth through the action of singing a song. Specifically, I infer that Matti first established his mom singing the birthday song once as the durational measuring unit of brushing his teeth. To Matti, the action of singing the birthday song had become a non-standard unit of durational measure that he could operate with to reason about the duration of his experiences.

Matti's explanation seemed to highlight three key aspects of his conception of duration as a measurable attribute of his experience: (a) conceptualizing time through activity; (b) creating

[2] I used [brackets] to show the participant's actions and hesitations.

spontaneous; durational units based on his experience; and (c) operating with these non-standard units as a means of durational measurement. Thompson's (1994) description of quantity seems to summarize these three distinct processes; "Quantities are conceptual entities. They exist in people's conceptions of situations. A person is thinking of a quantity when he or she conceives a quality of an object in such a way that this conception entails the quality's measurability" (p. 7). However, how young children reason about such quantities may be more reflective of Piaget's (1965) description of gross quantity, or the beginnings of quantification (as I will explicate in Chapter II). I will briefly situate each of these quantitative conceptions within the current field of time literature and research as the foundation of my conceptual framework.

Conceptualizing Time Through Activity

Recognizing time through human experiences is much like Einstein's relative spacetime, where "time and space form an inseparable whole" (Russell, 2008, p. 6). In this way, an individual can only acknowledge time by the experiences that fill it (Piaget, 1969). Even physical markers of passing time, such as Earth's natural revolution, only become a temporal event (i.e., 24-hour day) because people have recognized them as such. Thus, the filling of space characterizes humans' construction of time and creates its measurability.

Reisman (1971) explained that, "the measurement of time employs the counting of frequencies to indicate duration (with frequencies here referring to the number of temporal occurrences, such as daylight and darkness). This might also be described as 'an operation of counting units and subdividing them'" (Clémence, 1966, p. 404, as cited by Reisman, 1971, p. 152). These operations are like Thompson's (1994) description of quantity. In talking about the experience of teeth brushing, for example, Matti took the durational experience (between that

start and end of his brushing activity) and unitized it as the active combination of listening to his mom singing the ABCs and him spitting the toothpaste out.

It seems that Reisman assumed "the counting of frequencies" or "counting units" is an understood mental action. But one is left pondering, frequencies of what? Units of what? Matti seemed to recognize these durational frequencies through his actions, thus units of duration were units of experience (e.g., hearing the song and spitting in the sink). I will now elaborate on how a child might come to create such "make-shift" (Piaget et al., 1960, p. 27) units.

Creating Durational Units

Von Glasersfeld (1981) postulated an attentional model to explain the conceptual construction of unit as a basic capacity of the mental system. To this end, he defined a*ttention* as a brief, pulse-like awareness of sensory-motor input, bounded by unfocused pulses (von Glasersfeld, 1981). Von Glasersfeld was particular in asserting that this awareness cannot extend over longer durations, but rather is "moments of attention" (von Glasersfeld, 1981, p. 85). When considering duration, these *moments of attention* can "bound" an experience and allow for durational measurement of the experience. For example, the guest from the baby shower was aware of the noon start time of the event and of the time she read on the clock; however, she seemed unaware of the passing time as she (mis)judged the duration of her experience. Von Glasersfeld called this "*perceptual framing*" (p. 86).

During any experience, an individual must either purposefully (e.g., focusing on someone who is speaking) or by happenstance (e.g., being startled by an unexpected bell) focus their attention. Von Glasersfeld (1981) gave the example of a cocktail party, where individuals overlook background noise to attend to a particular conversation. "We *do* divide our visual, auditory, and tactual fields of experience into separate parts which, in our cognitive organization,

then become individual items or 'things'" (von Glasersfeld, 1981, p. 86). Von Glasersfeld called these "things" *unitary items*. Drawing from Thompson's (1994) description of quantity, individuals might consider experiential unitary items as the object from which they could quantify the quality "duration."

Unitary items come to be by being distinguished from their surround in one's experience. In other words, there must be something to differentiate them from the rest of the experience. When reflecting on duration, however, all the sensorimotor items are inherently bound, creating the single experience. For example, for Matti, brushing his teeth was not just the action of brushing and spitting, but also the sound of his mother singing, perhaps the taste of the toothpaste or the fatigue he felt in his hand or arm, and so on. By disregarding some aspects in his field of experience, an individual could quantify duration through the cognitive organization of these sensory items creating a single experiential unit. What might differentiate one experiential unit of duration from another would be an interruption in space (e.g., the start of a new task or a change in position).

Because all perceptual items are part of the overall experience, it seems reasonable that only a spatial interruption can distinguish durational unitary items—such as a bell ringing or completing a task. For Matti, this interruption might have been his mom finishing her song. These occurrences conclude ("bound") the durational experience as it was, causing a new durational experience to begin, such as Matti spitting out his toothpaste or leaving the bathroom. Each experience, then, can then become a unitary durational experience that can be quantified. I will now turn to how an individual might use such durational units to measure time, as described by Long and Kamii (2001).

Durational Measurement

Long and Kamii (2001) explained that time measurement requires three logico-mathematical operations, which they termed conservation of speed, transitivity, and unit iteration. *Conservation of speed* means that time and the units to measure it are uniform. Matti established a unit of singing the ABCs to measure his experience of brushing his teeth. In his explanation, it seems that he conceived of this experience as being an unchanging "thing," or unit, that others would also understand as such. Those units he supplied further accentuate their presence in his experience of two critical boundary occurrences: the start of brushing, which he aligned with the start of his mother's singing, and the end of brushing then spitting out his toothpaste, which coincided with completion of his mother's song.

Transitivity, as previously explained, is the act of comparing quantities against one another. Because, to Matti, singing the ABCs had become a unit of durational measure, he could use it to make a comparison against the act of brushing his teeth. *Unit iteration*, as described by Long and Kamii (2001), refers to iterable units of one (Steffe, 1991), where a single unit is repeated across a whole. For Matti, only one unit of the ABCs created the duration of his entire experience. The two activities were of equal length, thus there was no repetition this unit. How Matti might consider this unit to measure the duration of other experiences may demonstrate unit iteration as defined by Long and Kamii (2001).

Long and Kamii (2001), Piaget (1969), and Earnest (2019), among others, emphasized that the most challenging aspect of durational measurement is the need to abstract this critical facet of time. Because time itself is intangible, the units used to measure time must also be intangible, thus must be constructed within the mind of the individual, similar to Thompson's (1994) consideration of quantity as a conceptual entity. Accordingly, the primary focus of this

study is to look at the units that children might construct when considering the measurement of time. The review of literature in this chapter pointed to the mind's momentary attention to occurrences it takes as "start" and "end" of an experience as the source for such abstraction.

Research Question

This study explored the markers (Tzur, 2019) children might have as they conceptualize duration as an attribute of their experiences prior to formal, school-based time instruction. My review of the research literature on children's development of time reasoning suggests that past research has been focused on five major areas. The first area focuses on children's abilities to read and interpret clocks (Boulton-Lewis et al., 1997; Burny et al., 2009, 2012; Earnest, 2017, 2019; Korvorst et al., 2007; Meeuwissen et al., 2004; Metelerkamp, 2013). The second focuses on children's use (and learning) of the language of time (Earnest, 2015; Evans, 2004; Tillman & Barner, 2015). The third area focuses on children's perceptions of durations, including the role of emotion and attention to perceived lengths (Droit-Volet, 2011; Droit-Volet & Coull, 2015; Droit-Volet & Wearden, 2016; Wearden, 2016; Zakay, 1992). The fourth focuses on children's qualitative time reasoning (Kamii & Housman, 2000; Kamii & Russell, 2010, 2012; Levin, 1977, 1979; Levin & Gilat, 1983; Levin et al., 1978; Long & Kamii, 2001; Piaget, 1969; Richie & Bickhard, 1988; Russell, 2008; Russell & Kamii, 2012; von Glasersfeld, 1996). The fifth focuses on formal time instruction (Burny et al., 2009; Burny et al., 2013; Nelson, 1982; Reisman, 1971).

Given this vast body of research, I found no studies on what conceptualizations a child might have of duration as an attribute of their lived experience. Accordingly, my study addressed the following research question:

When reflecting on past experiences, what conceptions do children (grades preK-1, ages 4-6 years old) seem to have about the duration of those experiences, as indicated by their descriptions of duration?

CHAPTER II

LITERATURE REVIEW

> Even though the field has come to value the importance of children's…everyday experiences in other areas of mathematical measure (e.g., length; Lehrer, 2003) …time measure has been reduced to clock reading and calculations of elapsed time. For children, time seems to reflect procedural problems encountered in school settings rather than a conceptual tool to reason about properties of the world around us. (Earnest, 2018a, p. 1-2)

The theoretical basis for this study, presented in Chapter I, provided contextual background of how some Western theorists have conceived of time throughout history. Most of these individuals worked to classify and define what time is but not necessarily how it is understood. Aristotle (300BCE), for example, imagined time as a type of number, while Hume (1737/2001) explained time through the successions of perceptions, and Kant (1781/2007) contended that time derived from experience. Piaget (1969), on the other hand, took Einstein's (1920) concept of space-time as the basis for a comprehensive investigation on how children conceived of time. His foundational text, *The Child's Conception of Time* (Piaget, 1969) laid epistemological bases for the field of time study in mathematics education research.

My research purpose was to begin identifying conceptual markers (Tzur, 2019) of young children's conceptions of duration as an attribute of their lived experiences as they mentally *re-present* (von Glasersfeld, 1991) these experiences. By re-present, I mean how a child mentally recalls and re-runs the activities of their experience during their reflection on specific durations. In focusing on such re-presentations, I follow Piaget's (1969) definition of time linked to the

memory of experiences, as *psychological time*. He postulated that "psychological time rests on two distinct and fundamental systems: the order of succession of events and the colligation of duration" (Piaget, 1969, p. 248). I interpret this to mean that the memories of past experiences have a recorded order of events, and an individual can retrieve, re-present, and unitize these events to conceive of them as the attribute of duration. To explain possible re-presentations that may underlie psychological time, I first provide my definitions of Piaget's temporal concepts—specifically succession, colligation, and duration—within the context of my research question. Then, I provide further review of more current literature on children's conceptualization of duration.

Temporal Concepts

To re-present an experience, the individual must first record the experience (von Glasersfeld, 1995). The only system that an individual has available for such recording of one's experience is the brain (Medina, 2008). Furthermore, at the brain level, attention and processing is done singularly, as "multi-tasking" would cause processing limitations (Salo et al., 2017). Thus, as an experience is occurring, the individual's brain can either: (a) attend to and record the incoming sensorimotor information of the current experience (e.g., notice the sound of running water or the taste of the toothpaste); (b) re-present previous experiences (i.e., reconstructs past experiences); or (c) anticipate re-presentations of coming experiences (i.e., consider the series of activities of future experiences). Because any of these three mental actions can occur during a single sequence of experiences, an individual's recording of their experience is subject to their own attention while "in" the experience (Salo et al., 2017).

It is important to note that even in the first type of recording an occurring experience, what the brain registers cannot be considered "objective" in the sense that it fully coincides with

the sensorimotor input (Medina, 2008). An objective recording of the experience would require the use of a device that operates like a video camera. This would preserve the sensorimotor information without interference of attention. With such a recording device, the succession of events could be chronologically maintained and united (colligated or grouped) into a single experience-sequence. Such a chronological experience-sequence could become the root of objective duration and could be measured against standard or non-standard timing tools. Because such an objective recording is not possible by the human brain, the succession of colligated durations, which constitute Piaget's (1969) psychological time, is inherently subjective. That is, in my study a basic assumption is that succession, colligation, and duration themselves are necessarily subjective constructions of the human mind. Next, I further elaborate on each of these terms.

Succession

Succession is the re-presented order of recorded events. As explained above, this re-presentation may or may not coincide with the actual chronological order in which the events happened. According to Piaget (1969), succession has two facets. First, physical time is the child's perception of the sequence of events as they actually occurred. Second, psychological time is how the child reconstructs (re-presents) these events from their memory. The reflection upon past events in the re-presentation of memory occurs similarly in psychological and physical time (Piaget, 1969). Young children tend to recall narratives of experience in an intuitive jumble, hence their challenges with retelling stories in a logical order (Kintsch & van Dijk, 1978; Reed & Vaughn, 2011). For example, in February 2019, I had a conversation with a 4-year-old named Reece. I had asked Reece (pseudonym) how long she slept at night. Her response is shown in Excerpt 2.1.

Excerpt 2.1 (February 2019): Reece's Reflection of the Duration of Sleeping at Night through her Description of her Nightly Sequence of Events

1. R (Reece): I just take a shower [counts one finger][3] and I put my jammies on [counts the same finger again] and I just go to bed and I say good night to grammy and then I just watch TV in the middle of the night, that's what I actually do!

As she described her experience, Reece re-presented her "normal" bedtime sequence of events. However, her succession of events may not have coincided with the chronological order. For example, it seems unlikely that she watched TV after going to bed.

Over time, children's reasoning of succession begins to include consideration of the seriation of events, which at the operational stage may also include causality (Piaget, 1969). In Chapter I, I described a conversation I had with 4-year-old Matti about the duration of brushing his teeth. As Matti re-presented his experience, there was an implicit causality to his re-presented actions. For example, his experience may have begun as he put toothpaste on his toothbrush. This event preceded the action of actually brushing his teeth, because the logical progression of activity (Piaget, 1969) would not have been to brush his teeth and then put toothpaste on his toothbrush. It is important to note here that such a logical inference might not have been explicit for him; it is I, as an adult researcher, who attributed it as a possible facet of his experience. He bound his experience after his mom finished singing with spitting out his toothpaste. More specifically, Matti's re-presentation of his experience was bound by the start and end points of *his* combined (colligated) actions.

[3] I used [brackets] to show the participant's actions and hesitations.

Colligation

Piaget (1969) described *colligation* as the organization of successive durations. This grouping is the means through which "duration acquires a structure" (p. 159). This serial grouping of durations is innately successive given the sequence of re-presented events. For example, the steps Matti may take when brushing his teeth include successive, partial durations—the actions of putting on toothpaste, brushing his teeth, and rinsing his toothbrush—into the overall duration of "brushing his teeth." Because colligation involves the child's recognition of part-whole relationship of partial durations and the overall duration of a re-presented event, Piaget (1969) posited that children who can operationally reason about the colligation of durations could understand the homogeneity and continuity of time.

Homogeneity is the understanding (abstraction) that time is not localized, but rather co-occurs for everyone. Russell (2008) illustrated this with the homogeneous time of children at school and their parents at work as existing simultaneously, despite the different events that filled it. In other words, the child, or parents' idiosyncratic durations are considered to co-occur in the overall duration of both. This duration of separately occurring events can be ordered and compared because of the *continuity* of time, that being "the metaphorical flow of time" (Russell, 2008, p 12). If time were to stop and start as one's actions changed, experiences could not be compared, and duration could not be quantified. For example, if a young child were to notice that their sibling had grown taller than their parent, they might infer that the sibling is older—equating age with height. Colligation of durations results because time continues in a homogeneous manner—such an abstraction provides the conceptual basis for it being measured.

Duration

While my framework draws on Piaget's (1969) theory, my definition of duration differs from his. According to Piaget (1969), duration is "a test of [an individual's] grasp of the order of events" (p. 35). I see duration as the sense of "being in" a re-presented, colligated succession of experiences, bounded by a starting and an ending point. This might allow the individual to quantify duration when re-presented, colligated successions are compared to one another. That is, "being in" a re-presented experience more or less than "being in" a different re-presented experience. Duration as an abstracted attribute of one's experience involves a comparison between two re-presented colligated successions. I, therefore, characterize *duration* as a comparative quantity involving a re-presented record of experience in relation to some other record(s) of experience. This, similar to Thompson's (1994) quantity, involves creating an appropriate (to the child) unit and the comparative process of assigning numerical value to the time of an experience.

Returning to my conversation with Matti—to quantify the duration of brushing his teeth, he compared his re-presented record of his mom singing the ABCs against his re-presented record of brushing his teeth. To do this, Matti had to bound (von Glasersfeld, 1981) his experience by attending to the starting and ending points of his actions. As he re-presented brushing his teeth, I infer that his reasoning includes an anticipation that the ending point would occur just after the ending point of singing the ABCs once. Such an attribution illustrates both the comparative nature of duration and how its consideration for being measured.

Qualitative Duration

With the definition of duration that I presented above, I consider *qualitative duration* similarly to Piaget (1969), which involves the re-presentation of partial durations within a larger

experience or the "whole" duration. For the individual, such partial durations are smaller than the experience precisely because they produced (or can create) the "whole." Matti re-presented brushing his teeth as a comparative duration between his actions and his mother's singing the ABCs. When considering the unit of the ABCs as a non-standard durational unit, Matti might consider how it compares to other experiences, such as washing his hands or eating his lunch. If he were to compare the ABCs against these other experiences, he might qualify them as being more (longer), less (shorter), or the same duration.

Regarding quantifying qualitative duration, Russell (2008) further characterized two types: *intensive quantification* and *extensive quantification*. I consider Russell's intensive quantification as analogous to Piaget's qualitative duration, as both involve the comparison of more than, less than, or the same as. Extensive quantification, according to Russell (2008), involves an individual's estimation of durational comparisons, such as "much more than" or "a little less than." Whether intensively or extensively (Russell, 2008), the basis of qualitative duration is that time intervals would be first quantified comparatively, not numerically.

Quantitative Duration

Russell (2008) termed *quantitative duration* as *numerical quantification*, with the use of units of measurement, and some form of numbers, as the basis of comparison. This, according to Piaget (1969), occurs when children distinguish space measurement from time measurement, (i.e., when a child can account for temporal duration without spatial comparison). This marks a noticeable distinction from Russell's definition, in that Piaget, when considering quantitative duration, is noting a conceptual distinction of space in which one's experience occurs from time (how long they last). When older children can estimate that it takes about two minutes to brush their teeth, they are reasoning quantitatively about duration (e.g., they can anticipate this would

be the duration of teeth brushing regardless of being done in one's house or at the grandparents' house). With such an abstraction, the child no longer needs to compare duration always in relation to another experience to explain its length. Rather, the child can conceive of it as a durational unit compared to another durational unit, including to the "same" durational unit (re-presentation of the "same" succession of experiences). This quantification is essential to anticipating, planning, and scheduling future events (Earnest, 2018b).

Measuring time in a way that changes from comparison of re-presented successions of colligated experiences to comparison with a durational unit relies on what Piaget (1969) called *uniformity*, or the consistent progression of time. This consistency underlies people's ability to create and use timing devices, such as clocks or hourglasses. Key to such use is the abstraction that time advances constantly—it does not speed up or slow down based on perception. Running faster, for example, does not make time go quicker; rather, it entails the duration between start and end become shorter. As Russell (2008) asserted, "[j]ust as a system of operational time must be constructed by each individual over time, the ability to quantify develops over time" (p. 13). An important note, whereas numerical quantification is the most advanced and abstract quantification of duration, it is discussed here to provide a larger context for the terms used by previous researchers—not as the focus of this study.

In summary, my study will explore children's conceptualization of psychological time (Piaget, 1969) that is, their coordination (colligation) of re-presented successions of occurrences into durations as comparative quantities of "being in an experience." Piaget (1969) is the only researcher to investigate the conceptual process of psychological time in children. Other researchers, however, have followed Piaget's theoretical framework with regard to physical time (Kamii & Russell, 2010, 2012; Levin,1977, 1979; Levin et al., 1978; Levin & Gilat, 1983; Long

& Kamii, 2001; Richie & Bickhard, 1988; Russell, 2008). Having characterized the key terms used in this dissertation study, I now turn to a more comprehensive review on such literature of children's temporal reasoning.

Literature on Children's Reasoning of Time

Piaget's Seminal Studies of Children's Conception of Time

Piaget's pioneering book, *The Child's Conception of Time* (1969), investigated many facets of temporal reasoning. A *conception of time* encompasses numerous temporal concepts: building from "*the elementary operations*" (p. 87) of time, i.e., the sequence of events and duration and advancing into physical time concepts, such as additive composition of durations and time measurement. Finally, Piaget discussed psychological, or inner, time. Across all concepts presented, through his clinical interviews Piaget explicated children's development from an intuitive to an operational conception of time.

One such investigation, for example, looked at children's conception of the equalization of synchronous durations. In this experiment, Piaget asked children to explain what happened as two differently shaped containers were simultaneously filled with identical streams of water. In other words, the durations of the bottles being filled were synchronized, but the height of the liquid appeared different. Explanations varied widely across the participants, and Piaget found a discernible correspondence between age and the stage of conception of time.

Children at the first, intuitive, state (between the ages of 4 and 5) were unable to explain duration qualitatively or quantitatively in relation to the water flow. These children conceived of time in relation to the apparent height of the water within the containers. For example, the tall,

skinny container that looked "more full" took longer to fill. This conception is similar to the notion that height and age are related, as described in Chapter I.

The next stage was split into two sub-stages (IIa and IIb), based on the children's ability to recognize and utilize the synchronization of filling of each bottle. Children at Stage IIa, most of whom were about 5 years old, could intuitively disconnect the height/duration correlation of the first stage. That is, they seemed to recognize that the bigger container would take longer to fill even though it would look less full during the same duration. These children, however, could not recognize that given the same start and stop time of the flow, the fill of the containers must necessarily have the same duration.

Conversely, children at Stage IIb (ages 6 and 7) were able to recognize the synchrony of the fill through guided questioning and, once recognized, could apply the concept to other situations. Piaget (1969) called this activity, an "empirical discovery of synchronization" (p. 132). I link this understanding via prompting with a plausible *participatory* stage (Tzur & Simon, 2004) of the equalization of synchronous durations, where "knowledge is only available…in the context of the activity through which it was developed" (p. 296).

Piaget based the final stage (III) on the children's automaticity of the synchronicity and length of duration. Children at this level (all were 7 years old) were able to immediately recognize that, given the synchronization of the water, the duration of the event had to be the same. These children answered questions about the scenario with little explanation, but rather gave simple matter-of-fact statements, such as "the same time because the same amount of water was run" (Piaget, 1969, p. 137). According to Piaget, these children had established the concept of synchronization of duration, and they no longer needed the process of neither discovery nor confirmation to understand. I link children at this stage to an *anticipatory* stage (Tzur & Simon,

2004), in that they do not need to run through the activity to conceive of the equalization of the durations—the children quantify the durations before/without any activity (or prompting).

The equalization of synchronous durations is foundational for combining, comparing, and measuring durations (Piaget, 1969). Considering that duration is the quantity that time accounts for, and that Piaget found foundational conceptions of duration to develop as objects around age 7—roughly the age of second graders—it seems incongruent that formal, school-based instruction on time begins in first grade according to the Common Core State Standards Initiative (CCSSI: Measurement and Data, 2020), as highlighted in Chapter I. Much of Piaget's (1969) work laid the groundwork for future studies of children's temporal reasoning, particularly the work of Constance Kamii and colleagues (Kamii & Long, 2010, 2013; Long & Kamii, 2001).

Kamii's Studies on the Development of Operational Conceptions of Time

Kamii, a protégé and colleague of Piaget, drew on his constructivist framework, and conducted numerous studies on children's operational time reasoning. These included a focus on transitivity in the measurement of time (Long & Kamii, 2001) and calculations of elapsed time (Kamii & Russell, 2012). Kamii's work with Long (2001) and Russell (2010; 2012), as well as Russell's (2008) dissertation, replicated and extended many of Piaget's (1969) studies.

None of Kamii's studies focused specifically on children's conceptualization of duration. Yet, these studies greatly informed my thinking about Piaget's (1969) findings, about children's temporal reasoning, and about the incongruity between current school-based instruction on time and children's operational reasoning. Additionally, because Piaget's (1969) research came from French children in the mid-1920s, Kamii and colleagues recent work with American children more closely reflects the participants in my study.

Long and Kamii (2001) examined children's measurement of time, specifically looking at transitivity and unit iteration. I note that this deviated from Piaget (1969), who explored transitivity of duration as a more advanced conception deriving from the conception of operational time. Long and Kamii, however, used musical clips of varying lengths to investigate children's reasoning from kindergarten, through second and fourth, to sixth grade. The authors asked the children to compare the length of two pieces of music using various measurement materials. Expectedly, none of the kindergarteners were able to measure or compare the durations accurately. With increase in participants' age (grade-level), their ability to complete the tasks also increased. Only about half of second graders were able to measure and compare the durations of the musical pieces using non-standard units of time measurement. This is consistent with Piaget's findings discussed above. More importantly, this draws into question how the creators and implementers of CCSSI imagined first graders to be able to use standard units of measure to tell time on both analog and digital clocks (CCSSI: Measurement and Data, 2020).

Additionally, Long and Kamii (2001) asserted that children most easily measure length, then volume, and finally time, since time is an abstract unit that cannot be physically and continuously detected the way other forms of measurement can (p. 168-169). Their findings, again, pointing to the importance of continued study of children in those young ages. As I discussed in Chapter I, CCSSI includes standards to support the development of children's two- and three-dimensional measurement reasoning, but no such standards exist for time.

Kamii and Russell (2010) replicated an experiment by Piaget (1969) on children's development of operational time by focusing on the child's ability to distinguish age (time) from height. The authors found that children's intuitive reasoning to equate height and age persisted a year later than Piaget (1969) had asserted, to the age of 8 or 9. These findings indicate that

qualitative operational time conceptions may not develop until at least third grade—two years after children are first introduced to time telling/clock reading by CCSSI (CCSSI: Measurement and Data, 2020).

Later, Kamii and Russell (2012) specifically addressed the discrepancy between CCSSI and time reasoning in a study of elapsed time. Once again drawing from Piaget's (1969) work, the authors explored how children coordinated standard units of time (minutes and hours) to determine elapsed time. While Kamii and Russell (2010) illustrated that children were able to qualitatively infer elapsed durations by third grade, the ability to numerically quantify elapsed time is a more advanced conception that develops later. According to the authors, the understandings necessary to measure elapsed time are the same as those necessary to comprehend our place value system. Because the hours and minutes are on two "hierarchical" levels, much like tens and ones, the units coordination needed to account for passed time, as a single quantity is high. This ability did not fully develop in children until sixth grade (Kamii & Russell, 2012), though it is a third-grade standard according to CCSSI (CCSSI: Measurement and Data, 2020).

In her dissertation, Russell (2008) looked specifically at children's reasoning and compared it against Common Core expectations. Building from clinical experiments of both Piaget (1969) and Long and Kamii (2001), Russell examined children's "operational system of time" (p. 11). Russell had her participants complete Piaget's (1969) previously mentioned durational reasoning experiment, observing a consistent flow of water through various shaped containers to determine if the duration of flow was equal despite the inconsistent shape of the water (the bottle). Piaget's (1969) results found children's construction of this type of logico-mathematical reasoning around the age of 9; however, Russell found it to be appreciably later. In

Russell's own words, "The sixth graders in my study ranged in age from 11 years and 4 months to 12 years and 5 months, and yet the majority of these children did not have an adult understanding of time!" (p. 61-62). Simply put, Russell found that children beyond the elementary (K-5) grades were unable to reason logico-mathematically about the synchronicity of durations. Based on these findings, it seems that CCSSI recommends teaching children clock reading and time telling years before developing the necessary conceptualization of *what it is* that clocks are measuring.

Kamii and colleagues employed many of Piaget's (1969) studies using a similar conceptual and experimental framework. In their studies, the use of logico-mathematical knowledge was fundamental in the explanation of the children's performance. That is, they showed that a lack of logico-mathematical knowledge explained a lack in time reasoning. Though Kamii identified variations in the ages of certain benchmarks established by Piaget (1969), the mental processes she identified remained consistent with Piaget's research.

Not all researchers, however, followed as closely to Piaget's (1969) methodology or general findings on children's conception of time. I will now describe a few such studies, as they described children's durational reasoning.

Other Studies of Children's Durational Reasoning

Several other researchers have studied children's development of durational reasoning since the 1970s, including Levin and colleagues (1977, 1979; Levin et al., 1978; Levin & Gilat, 1983) and Richie and Bickhard (1988). Levin's work directly challenged Piaget's (1969) studies of children's conception of time, and Richie and Bickhard (1988) contended both Piaget and Levin's work.

Levin's various studies refuted the ages proposed by Piaget's (1969) findings, as well as the structure of his interviews. Levin (1977) suggested that Piaget's experimental designs could lead to deceiving results because of the spatial components of the experiment itself. Levin believed that "the child's conceptions of time are basically temporal but are vulnerable to counter suggestions from various factors, including spatial variables" (p. 442), such as distance covered or rotational versus linear movement.

To address these "counter suggestions," Levin and Gilat (1983) explored children's conceptions of duration using the intensity of a light, rather than the motion of an object. They found that young children viewed intensity as synonymous with duration (i.e., the higher the intensity, the longer the duration). Piaget, they believed, had not addressed this. However, I think that rather than supplementing Piaget's theory with the notion of "counter suggestions," these findings support Piaget and indicate a lack of operationalized durational reasoning. My rationale is that in both lines of work the children could not distinguish the two concepts, similar to previously mentioned age/height distinction.

Levin and colleagues (1978) also challenged Piaget's (1969) research design and findings. They asserted that durational reasoning was more difficult than reasoning about succession and would therefore develop later in the progression of time concepts. Piaget (1969) claimed that these two concepts developed simultaneously and interdependently. From my description, to quantify duration one first needs to coordinate (colligate) a succession of events to compare to another colligation of events. That is, I assert that a conception of succession is a prerequisite to durational reasoning, similar to Levin and colleagues' assertion.

Additionally, Levin and colleagues (1978) also argued that Piaget's experimental designs were misleading. They believed that by using speed and distance in his experiment, Piaget

created “distracting variables” that confused his participants. Instead, the authors focused their study on speed only, which, they argued, allowed for a more accurate demonstration of succession and duration separately. Levin (1979) continued to study the time/speed relation; however, in this study, Levin corroborated Piaget’s findings that children at the intuitive level of time reasoning confused the speed of an activity with its duration.

For the most part, Levin’s research seemed focused on disproving Piaget’s methodology and findings of children’s development of operational time. This, according to Russell (2008), was due to a lack of understanding by Levin and colleagues about the logico-mathematical nature of time concepts. Russell went on to cite specific instances of this impression, such as Levin’s insistence on separating succession and duration. According to Piaget (1969), the ability to coordinate these two concepts is precisely what characterizes operational time reasoning. Thus, Russell claimed, isolating the two creates a lack of logico-mathematical reasoning. This, Russell argued, limits a child’s reasoning about time to intuitive perceptions only—perhaps explaining Levin’s (1979) support of Piaget’s conclusions about children’s intuitive confusion of speed and time.

Richie and Bickhard (1988) also challenged the work of Piaget (1969), as well as the methodology and findings of Levin (1979). In their study, based on Levin and colleagues’ (1978) and Levin’s (1979) models of duration via light intensity, Richie and Bickhard questioned whether presenting more varied durational comparisons would allow young children to make “correct relative time judgments” (p. 319) based on non-inferential, perceptual cues. For example, rather than creating scenarios with narrow comparative durational ratios using intensity (e.g., a bright light for four seconds versus a dim light for seven seconds, as used by Levin and colleagues), the authors introduced more dramatic durational differences with the same light

intensity, such as one second versus seven seconds. Akin to Russell's (2008) critique of Levin's removal of "nontemporal cues," by claiming that succession and order of events were "removed" from duration, no coordination was needed, thus no logico-mathematical reasoning took place. Additionally, by claiming succession and order of events are not inherent in all durational experiences, Richie and Bickhard (1988) redefined time and how perception of time can be explained. I question what it means for children to perceive time non-inferentially, if time itself is an intangible, abstract concept, or, as Earnest (2019) called it, an "invisible quantity."

Earnest's Reified Units of Time

Earnest (2017, 2019) explained children's construction of temporal units through the theory of reification (Sfard, 1994). He contended that, "Time eventually became reified as length-based units that, consistent with the accepted structural conventions of number lines or other linear scales, reflected mathematical properties of unit and interval" (Earnest, 2019, p. 2). Considering the focus of this dissertation, I note that Earnest's focus was on how children used these measures to reason about duration using analog clocks, not the process by which these units might have become reified.

Earnest (2017) investigated how clock type influenced children's ability to reason about elapsed time, or "the duration of an event" (p. 195). He presented second and fourth grade children (approximately 7 and 9 years old, respectively) with three different clock tools: a digital clock, an analog clock with linked hands (meaning that the hour hand moved in relation to the minute hand), and an analog clock with independent hands. Each child was shown a "start time," then told an elapsed duration and asked to show the end time on the clock. Earnest found that clock type influenced if the child got the "correct answer," but not necessarily how the child reasoned about the duration.

Earnest (2017) also asserted that, "Telling time on an analog clock is…consistent with the treatment of geometric intervals and numeric units of number lines" (p. 193). To theorize "units of number lines," Earnest employed units coordination as explained by Steffe (1992, 1994, 2001), Hackenberg (2013; Hackenberg & Tillema, 2009), and Izsák, Jacobson, de Araujo, and Orrill (2012). Specifically, Earnest defined units coordination as "distributing a composite unit across the elements of another composite unit" (Steffe, 1992, p. 279, cited by Earnest, 2017, p. 192). This definition, however, refers specifically to the multiplicative coordination of two units. Units coordination is a more general construct, referring to the ability "to create units and maintain their relationships with other units that they contain or constitute" (Norton et al., 2015). And while the units coordination needed to reason about duration by means of analog clocks may necessitate two levels (e.g., a compilation of hours composed of a composite of minutes, as noted by Kamii & Russell, 2012), construction of spontaneous, non-standard durational units seems more likely to begin with a single level of units.

In 2019, Earnest continued exploring how elementary-aged children reason about duration via analog clocks by investigating how they might come to "see" intervals of time. Time intervals, Earnest asserted, maintain consistent "and invariant standard units…but these might not be the ideas salient to children grappling with a clock's structure as they reason with what experts see as time intervals" (p. 4). This is an important distinction that children do not "see" what adults "see," which I will more fully discuss in my conceptual framework.

In his study, Earnest (2019) described second grader Mela's reasoning about time intervals on an analog clock. Earnest cited, for example, Mela's explanation of a "'weird'-ness that 9:15 could be 30 minutes after 9:10" (p. 23) as evidence of her experiences of time across a day. This "weirdness" seems, to me, to be about the units on the number line of an analog clock

described by Earnest (2017), not necessarily about Mela's durational experiences of 30 minutes. Earnest explained, however, that Mela's verbiage impacted his interpretations of her reasoning. When explaining her placement of 9:15 as 30 minutes after 9:10, Mela repeatedly stated "it's already 9:10" (p. 17). Earnest contended that the word *already* "communicates one position in time relative to the another" (p. 17). He cited the linguistic studies of Tillman and colleagues to support his conclusion.

Tillman and Barner's (2015) Linguistic Study of Durational Words

Unlike the other studies discussed, Tillman and Barner (2015) investigated children's conceptions of duration from a linguistic rather than logico-mathematical perspective. The focus of their study was on word meaning, which the authors called *interpretation*. I believe that there is an inherent disconnect between the notions that children's interpretation of duration words is somehow different than their quantification of the durations. For example, Tillman and Barner asked children to organize day, hour, minute, and second on a timeline. The essence of this question is not linguistic, but rather quantitative in the relations between the different time units, as previously explained by Steffe's (1992) notion of composite unit.

The overall method of the interviews followed much the same design as those already mentioned. The authors interviewed individual participants in three separate experiments. Experiment One focused on how children from age 3 to 6 compared standard time units (e.g., I jumped for 1 hour and you jumped for 1 second, who jumped longer?). Experiment Two built from the findings of the first in that by the age of 4, many of the children understood the durational "rankings" of the time units/terms (i.e., an hour is longer than a minute, which is longer than a second). However, in Experiment One, the oldest participants (age 6) were not all able to organize the units according to these relative magnitudes. Thus, Experiment Two

explored how children from the age of 4 to 7 compared composite time units (e.g., 3 minutes versus 2 hours). For both experiments, the authors gave no evidence of inquiring into the children's reasoning about their quantification of the durations.

The final experiment differed from the first two in that Tillman and Barner included adult participants as a comparison. Children (age 5, 6, and 7) and a group of adults were asked to organize durations on a timeline of sorts. The participants arranged numbers (4, 25, 18), familiar events (washing hands, eating lunch), time units (minute, day), and composite time units (3 hours, 5 minutes) on four number lines, respectively. Across all three experiments, the ability to organize durations increased with age. Interestingly, not all adult participants showed multiplicative relationships between durations. For example, the location of 9 minutes was not necessarily at a distance that is three times larger than 3 minutes. Likewise, the relationship between 9 minutes and 2 hours showed no correlation to actual time measurements (e.g., the distance from the start to 2 hours was twice as much as start to 9 minutes). Again, the lack of information of how the participants reasoned made it unclear how the participants conceived these durations.

Having reviewed the growing body of research on children's reasoning with time, I could only find one study that investigated how children characterize time for themselves. Berggren (2018) presented preliminary findings on how children created pictorial metaphors for time, without the need for language to define the abstract nature of the concept. Next, I turn to that study.

Berggren's Investigation of Children's Metaphors for Time

Berggren (2018) had 47 Swedish third graders (age 8 and 9) draw a picture of what time meant to them. The author coded the children's visual representations into three themes: (a)

Time as an external phenomenon, such as measurement on clocks or the rising and setting of the sun; (b) Time as a subjective experience, such as the perception that it can move faster or slower depending on the activity; and, (c) Time being abstract and existential, such as time being infinite or created by humans. Berggren's study informs my dissertation in that it began with children's own conceptions, whereas other researchers situated their studies in their own understandings of time, for example, time in relation to space (Kamii & Russell, 2010; Long & Kamii, 2001; Piaget, 1969) and time associated to clocks (Earnest, 2015, 2017, 2019). In a sense, Berggren's (2018) study seemed to take as an important starting point the distinction between an adult's and a child's way of reasoning – a basis for creating second-order models of the latter.

It seems that much of the existing research on children's reasoning of time has disregarded second-order models (Steffe, 2000), meaning that researchers seemed to implicitly expect that children's understandings about time began from the same place as their own (e.g., clocks or standard units of time). However, time—being an abstract concept—must be created within the mind of the individual, and cannot be assumed by the researcher (Piaget, 1969). Beginning with the child's conception of time, like Berggren (2018), seems a logical starting point to explore children's conception of time as an attribute of their experiences. This aligns with my conceptual framework and provides an entry point for exploring children's conceptions of psychological time.

In summary, thus far I have provided background on temporal concepts necessary for reasoning about psychological time, specifically succession, colligation, and qualitative and quantitative duration. Additionally, I presented an overview of literature on children's conceptions of time that influenced my research. Specifically, I described Piaget's (1969) exploration of children's conceptualization of time, and that of Kamii and colleagues (Long &

Kamii, 2001; Kamii & Russell, 2010, 2012; Russell, 2008) who built specifically on Piaget's (1969) work. I also reviewed research that questioned Piaget's (1969) studies (Levin, 1977, 1979; Levin et al., 1978; Levin & Gilat, 1983; Richie & Bickhard, 1988) and pointed out to what I see as theoretical flaws in their arguments. Finally, I described some recent research in children's temporal reasoning, including Earnest's (2017, 2019) studies of analog clocks and elapsed time, Tillman and Barner's (2015) linguistic studies of durational words (which I draw from in my interviews, see Chapter III), and Berggren's (2018) investigation of children's metaphors of time (which I also drew from when designing my Methodology). Next, I explicate my conceptual framework, situating my assumptions about thinking from a constructivist lens and founding my upcoming Methodology in Chapter III.

Conceptual Framework

When considering how children conceptualize and operate with duration, past research has utilized theoretical lenses rooted in constructivism (Kamii & Russell, 2010, 2012; Long & Kamii, 2001; Piaget, 1969; Russell, 2008) and in sociocultural theories (Berggren, 2018; Earnest, 2015, 2017, 2019). I root my study in the assumption that children come to conceptualize duration through experience (Dewey, 1938; Piaget, 1969, 1971; von Glasersfeld, 1995), with a focus on cognitive facets of experience. That is, I consider experiences as events occurring within an individual's cultural context. These events are planned for, gone through, and reflected on within the minds of the individual. Accordingly, I used a constructivist framework for this study.

My study of children's conceptualization of duration as an attribute of their experience thus draws on radical constructivists' (von Glasersfeld, 1995) stance that knowledge does not exist independent of the learner, nor is it "out there" to be discovered. Instead, knowledge resides

in the mind of the individual and is constructed based on the individual's own experiences (Piaget, 1985, 2001; Steffe, 1995; von Glasersfeld, 1995). Dewey (1938) asserted "a person…gets out of his present experience all that there is in it for him at the time in which he had it" (p. 49). While this assertion was not specific to temporal reasoning, Dewey's meaning still seems pertinent regarding a person's attention. So, for example, a first-grade student who is beginning to learn clock reading (CCSSI: Measurement and Data, 2020) is basing their understanding of the units of clock measure on past experiences with clocks or standard time units. In this case, American idiomatic phrases, such as "just a second" or "hang on a minute," may underlie the child's early conception of the temporal units second and minute (Evans, 2004; Tillman & Barner, 2015). The problem with such experiences is that the context in which the phrases use these standardized units is not at all standard. Consequently, the first-grader's early conception of seconds and minutes may depend on linguistic interactions corresponding with general, imprecise durations (Tillman & Barner, 2015), not experiences of iterating standardized temporal units.

To characterize what durational conceptions children might have, it is important to consider the types of knowledge available and the construction of such knowledge. In this section, I first describe durational reasoning across three types of knowledge: physical, social, and logico-mathematical (Long & Kamii, 2001; Piaget, 1971). I then explicate the general construction of conceptual schemes (Piaget, 1985; von Glasersfeld, 1998) with emphasis on von Glasersfeld's (1998) explanation of scheme as a three-part phenomenon and Piaget's (1971, 1985) assimilatory schemes. Finally, I turn to how an individual might unitize duration, as introduced in Chapter I.

Types of Knowledge

Physical knowledge is a conceptualization of attributes that an individual might recognize through observation (sensorimotor perception). When considering time, such knowledge might include recognizing the numbers and position of the hands on an analog clock, such as the guest at the baby shower from Chapter I. This type of knowledge is purely about the individual's perception of physical objects, rather than the meaning individuals impart on the objects (Long & Kamii, 2001). In other words, for the guest at the baby shower, her physical knowledge about the clock was not the same as her reasoning about the duration of her experience.

Social knowledge is the conventions constructed by people, such as how clocks are read (Long & Kamii, 2001). Korvorst and colleagues (2007), for example, presented a study of the efficiency of naming time in relative (half past three) or absolute (three-thirty) terms across various cultures. In this case, the language of time is social knowledge. Because all human constructs are contextual (Vygotsky, 1978), social knowledge varies from place to place. In the United States, for example, clocks and time are closely interconnected, as much of our social milieu seems driven by the clock (Levine, 1997). This particular social knowledge vastly overgeneralizes American culture, as documented by Levine (1997) (i.e., "laid back Southern-Californians" versus "Type-A New Yorkers" [p. XIX]), which is a crucial factor when considering how different children might represent time.

Logico-mathematical knowledge is the mental relationships constructed through reflection on and abstraction of one's (re-presented) experiences (Piaget, 1971). For example, when a child is able to reason about the inverse relationship between motion and time (i.e., to take less time they must move faster), or that the hour hand on an analog clock moves in relation to the passing minutes, they are reasoning with logico-mathematical knowledge. The guest at the

baby shower reasoned logico-mathematically when she compared her perceived experience against the time she misread. Even though her quantification of the elapsed time was inaccurate, her mental action of comparing was logical. How the woman *assimilated* (or made sense of) her situation can be explained through inferences about her schemes.

Schemes

Piaget (1985) posited that an individual processes experiences through the mind automatically based on what they already knew—these mental structures he called schemes. *Schemes* are the basic cognitive unit an individual uses in assimilating, interpreting, reflecting on, and re-presenting their experience. Elaborating on Piaget, von Glasersfeld (1998) described scheme as a three-part mental structure. First, the individual perceives a given situation, which includes a specific goal, such as Matti looking for a way to quantify the duration of his teeth brushing experience. Second, the goal triggers a specific mental activity by which the individual's mental system would accomplish the goal. This activity might involve sensorimotor (physical) actions, internalized (mentally run-through) activity, or interiorized (conceptualized) operations. Matti, for example, interiorized the operation of comparing his re-presented brushing teeth experience against his activity of (iterating) the singing of the ABCs to quantify the duration. Third, the activity brings about a result that terminates the activity. Matti concluded (and quantified) the duration of his teeth brushing experience as equivalent to his re-presented action of the entire duration of his mom singing the ABCs.

Importantly, schemes are a researcher's lens through which they infer into a child's assimilation of and reasoning about their experiences. This is often different than the adult's own reasoning. For example, Matti assimilated my question, "How long does it take you to brush your teeth?" in a much different way than I had anticipated. My goal was for him to quantify the

duration of his experience—to tell me an amount of time. I had expected him to use some form of standard temporal units, such as "minutes" or "seconds," or to use general phrases of time, such as "a little while" or "not long." Matti, instead, interiorized the activity of his experience, and quantified the duration through another activity. His goal, and likely perceived situation (von Glasersfeld, 1995), was different than my own. This is much like the difference in perceived situations between the guest at the baby shower and my own. Both Matti and the baby shower guest quantified their experiences, but each did so in markedly different ways—and neither quantified their own experience, as I would have as a "third-party witness."

Von Glasersfeld (1995) described a perceived situation when an individual assimilates external information into their existing scheme. The individual's recognition template (von Glasersfeld, 1995) of their experience is the basis of this information, rather than on some "objective input" or others' view of the situation. Because no two perceived situations are identical (Medina, 2008; von Glasersfeld, 1995), how two people re-present, or reconstruct, the same experience will be different. For example, when I asked 4-year-old Reece how long it took her to brush her teeth, she hesitated and stammered, then responded, "5 minutes." (I note that, as a researcher, I did not take her utterance of "5 minutes" as a reflection of understanding the meaning and/or duration involved.) Critically, Reece and Matti were given the same goal—to quantify the duration of brushing their teeth—however, how they perceived the situation and the mental activity they carried out seemed different. I designed my study to mark (Tzur, 2019) those different ways in which young children may assimilate and reason about duration as an attribute of one's own experiences.

During the activity part of a scheme, an individual can use sensorimotor actions, internalized activity, or interiorized operations (von Glasersfeld, 1995). This activity allows the individual to recognize and manipulate constructs based on their prior schemes that are in use.

When an individual coordinates mental activity with kinesthetic actions performed on tangible objects, a *sensorimotor action* occurs (Norton et al., 2018; Piaget, 1972; von Glasersfeld, 1995). It might seem that when considering psychological time (past durational experiences) there can be no sensorimotor action because it would necessarily be physical time (in-the-moment durational experiences). However, I contend that when considering inner duration, sensorimotor actions might be re-presented situations that combine space and time in a single construct, resembling Piaget's (1969) intuitive time.

As I continued my conversation with Reece, I asked how long it took her to brush her teeth. Her response can be seen in Excerpt 2.2.

Excerpt 2.2 (February 2019): Reece's Reflection on the Duration of Brushing her Teeth

1. R: Um, maybe like [hesitates briefly] eh, 5 minutes.
2. A (Author): How do you know?
3. R: Uh [hesitates briefly] I don't know that while.
4. A: Is it a long while or a little while?
5. R: A little time.

Upon assimilating my question, Reece seemed unable to mentally re-present her experience of brushing her teeth. It could be inferred, therefore, that she could not internalize duration as an attribute of her experience; rather, her durational conceptions lay within the sensorimotor actions themselves. Reece's use of the duration "5 minutes" (Line 1) does not seem

to indicate re-presented actions, but rather some other conception in her existing schema—perhaps a linguistic experience arising from interacting with her parents.

An *internalized activity,* Olive (2001) asserted, occurs as "the child can mentally re-present the activity. This mental re-presentation still carries with it contextual details of the activity" (p. 4). As psychological time exists only within the mind of the individual, there is an inherent internalization when these experiences are reflected on. How an individual might internalize the duration of their experiences, however, can differ (as will be shown through my data analysis in Chapter IV).

Interiorized operations are the abstract coordination of previously internalized actions without the need for mentally "running through" the experience (Norton et al., 2018). In other words, internalization occurs when an individual can reason logico-mathematically to classify, seriate, or quantify a perceived situation (Kamii & Russell, 2012). This can be demonstrated through Matti's comparison of his re-presented experience of brushing his teeth against his re-presented experience of hearing the ABCs. Matti interiorized each experience and then effortlessly compared (coordinated) his re-presented experiences as he quantified the duration of brushing his teeth. Unlike Matti, Reece seemed to use a different re-presentation when later asked about her explanation of brushing her teeth versus eating her lunch.

Reece had explained that it took her 5 minutes to brush her teeth, which was "a little time," and that eating her lunch took longer than brushing her teeth. Yet, she went on to explain that it also took 5 minutes to eat her lunch. Interestingly, she said that the 5 minutes of lunch was longer than the 5 minutes of brushing her teeth. Reece's re-presentation seemed rooted in the internalized actions involved in brushing teeth and in eating lunch. At issue is not that she was unsuccessful in quantifying her experiences. Rather, it is her ability to re-present and compare

eating her lunch and brushing her teeth quickly and efficiently. I infer this indicates a pre-quantified comparison of sorts. It seems qualitatively different from Matti's measurement, and may serve as an initial distinction of variations in reflecting and re-presenting duration.

Von Glasersfeld (1995) described the last part of a scheme as "the expectation that the activity produces a certain previously experienced result" (p. 65). Matti recalled his mom singing as the duration of brushing his teeth. When he re-presented his experience, he seemed to draw from his existing durational scheme of "the ABCs" as a unit of measure to quantify his experience. In Chapter I, I identified three key conceptions that seem integral when exploring how a child might quantify psychological time, specifically, spontaneously creating non-standard durational units based on these activities, and operating with these units to measure the durations of experiences. I will now elucidate how each of these might lead to a quantification of duration, reminiscent to Matti's.

Creating Durational Units

Piaget (1969) asserted that "it is exceedingly difficult to get children to make spontaneous measurements of time" (p. 176), thus, his extensive study of children's conceptions of time did not explore how a child might come to create such measurements. Piaget (1969) did, however, explain that the creation of a durational unit requires creating a mobile unit that can be theoretically isolated from the temporal framework and iterated across a specified experience (p. 174-175). For example, when I asked Matti how long it took to brush his teeth, I was eliciting his possible creation of a unit of measure that he could use to quantify the duration of his experience. Matti conceptualized this measurement through the experience of hearing his mom sing the ABCs. Unlike Matti, a similar question did not elicit Reece's use of a durational unit, which is why I conjecture variations in how children might conceptualize duration.

Steffe (1991) asserted that a child could construct units from experiential items through a unitizing operation. In *Operations that Generate Quantity*, Steffe (1991) explained "Segmenting sensory experience into units is the result of a unitizing activity prior to measuring or to counting" (p. 63). With this, Steffe called attention to the importance of experience in the construction of units. To explain this construction, he drew on von Glasersfeld's (1981) *Attentional Model for the Conceptual Construction of Units and Number* as a model for a mental operation of unitizing. I build on this model when (see below) when inferring into children's re-presentation of their experience and thus duration as an attribute of that experience.

Von Glasersfeld (1981) founded his attentional model in neurological research that asserts that the brain cannot process all of the sensory material it receives at any given moment (Harter, 1967, as cited by von Glasersfeld, 1981; Tzur, 2011). Consequently, the brain utilizes a process called perceptual framing to "differentiate or 'cut' things out of a background and perceive each one of them as an entity or whole" (von Glasersfeld, 1981, p. 86). To explicate this unitizing mechanism, von Glasersfeld (1981) articulated:

> The hypothesis I am here proposing is that these unitizing operations consist in the differential distribution of focused and unfocused attentional pulses. A group of co-occurring sensory-motor signals becomes a 'whole' or 'thing' or 'object' when an unbroken sequence of attentional pulses is focused on these signals and the sequence is framed or bounded by an unfocused pulse at both ends. (p. 87)

For this study, I slightly modify von Glasersfeld's model by positing that simultaneous sensory-motor actions create a "whole" or "thing" or "object" of experience when successive attentional pulses are bound between two spatial interruptions. These interruptions act as a double boundary—bounding both the end of the current "whole" (experience) and the beginning

of the next. For example, Matti (Chapter I) began brushing his teeth at the same time as his mother began singing (one experience). When his mother finished the song, this seemed to end Matti's teeth brushing experience. He then described spitting, beginning a new experience. Through reflection within and across these bounded experiences, temporal unitary items may develop. The creation of such bounded items seems necessary for a quantified conception of reasoning about duration.

Unit iteration, as described by Long and Kamii (2001), refers to iterable units of one, where a single unit is repeated across a whole. Matti's reasoning that one completion of the ABCs would be equal to one complete teeth brushing seems to reflect his transitive reasoning but would not yet constitute unit iteration. If he were to take his unit of "ABCs" and repeat it across a different experience, say making his bed, to reason that it might take three ABC songs to equal one making his bed, this would be an example of such unit iteration. A child's creation of durational units seems inextricably bound with their construction of number. Next, I discuss how such a link might evolve from the development of children's numerical reasoning.

Children's Construction of Number

Pre-numerical

Steffe and von Glasersfeld (1985) described early number as a uniting operation, which develops through sensory-motor or other activity-based experiences, as previously described. Such experiences could be counting to 10 while playing hide-and-seek. At this stage, Ulrich (2015) clarified, children have constructed the count as the next word in the progression of counting (i.e., one, two, three, four, etc.), but not as a unit. According to Ulrich, the unitizing operation develops from perceptual material (an intuitive conception of time) and thus does not

define unit as an interiorized, usable thing. To understand number as a composite (a thing made of things), Ulrich stressed the need for reflection on activity (an operational conception of time).

Piaget (1965) defined this pre-numerical stage as gross quantity, or the "beginnings of quantification" (p. 10), where children perceive of asymmetrical differences, such as more or less. Ulrich (2015) described this as the construction of a *perceptual unit*. At this stage of temporal quantification, Piaget (1969) argued that psychological time and physical time are undifferentiated and therefore become "deformed" (p. 199). By deformed, Piaget is emphasizing intuitive temporal conceptions, such as taller people always being older. This suggests that, according to Piaget, children who reason intuitively about time do not, and cannot, quantify the duration experiences.

At the operational level, Piaget (1969) asserted, physical and psychological time can be differentiated—thus, can be quantified. To develop such an operational conception of time, the child would need to develop an *arithmetic unit* (Ulrich, 2015). The creation of such a unit would allow children to reason about duration apart from spatial relations in which it is bound. For example, if Matti were to operate with the ABC song to measure the length of other experiences, apart from brushing his teeth. When a child begins operating with such units (e.g., iterable units of one; Long & Kamii, 2001) they demonstrate a shift from pre-numerical to *Initial Number Sequence* (Steffe, 1992; Steffe & Cobb, 1988).

Initial Number Sequence

> We often think of units as a context for measurement…a second is a unit we can use to enumerate elapsed time. However, in order to conceive of being able to use a number to describe…magnitude…a child will use their counting numbers to find how many items are in a collection. (Ulrich, 2015, p. 3)

The Initial Number Sequence (INS) results from reflection on sensory-motor counting activities. It is the transformation of number from just the next word in the counting sequence to the accumulation of all successive units counted into the whole (Ulrich, 2015). To evaluate children's unit iteration, Long and Kamii (2001) had children use a timer to measure the length of a song. The duration of the song was longer than that of the timer, so the children had to keep track of the number of times they used the timer. A child distinguishing that the duration of the song was equal to the duration of four fills, might be said to have developed a durational INS. According to Ulrich (2015), such a process occurs when the child has interiorized a counting act and can abstractly anticipate the counting activity, producing the result of that counting activity. This is the basis of true durational measurement (Long & Kamii, 2001).

To Summarize

In this chapter, I described the temporal conceptions necessary for children to reason about psychological time, namely succession, colligation, and duration. I distinguished my own definition of duration from Piaget's (1969) by highlighting a necessary comparison of re-presented experiences as a means of quantifying duration. I presented a literature review of research on children's durational reasoning and temporal conceptions, including three studies specific to my upcoming Methodology: Piaget's (1969) foundation durational studies, Tillman and Barner's (2015) linguistic study, and Berggren's (2018) exploration of children's metaphors of time. I then explicated my conceptual framework, highlighting assimilation through scheme theory. Finally, I provided a framework that draws on conceptual models of measuring quantities (Piaget, 1969; Steffe, 1991; von Glasersfeld, 1981), which informs my Methodology (Chapter III) and Data Analysis (Chapter IV).

CHAPTER III

METHODOLOGY

> The aim of the [phenomenology] investigator is the reconstruction of the inner world of experience of the subject. Each individual has his own way of experiencing temporality, spatiality, materiality, but each of these coordinates must be understood in relation to the others and to the total inner "world." (Ellenberger, as cited in Hycner, 1999, p. 291)

In this study, I examined how children conceptualize duration as an attribute of their lived experiences. In following with a constructivist framework, as described in Chapter II, I strive to set aside my own experiences with time to understand how my participants conceived of duration prior to formal school-based time exposure.

This exploration built on the theoretical work of previous studies on children's durational reasoning (Berggren, 2018; Long & Kamii, 2001; Piaget, 1969; Tillman & Barner, 2015), with explicit focus on space-time relationship, durational transitivity, and unit construction in the measurement of familiar activities. Through this study, I also sought to expand upon the methodological framework for children's reasoning about time. In most previous time-related research, participants performed in-the-moment physical experiments (Earnest, 2017; Kamii & Russell, 2010, 2012; Levin, 1977, 1979; Levin & Gilat, 1983; Levin et al., 1978; Long & Kamii, 2001; Metelerkamp, 2014; Piaget, 1969; Richie & Bickhard, 1988; Russell, 2008; Tillman & Barner, 2015). In this study, instead, I had the participants reflect on their lived experiences (Berggren, 2018; Earnest, 2018a) to infer what Piaget (1969) termed *psychological time*. Specifically, my study of children's durational conceptions focused on the following research question:

When reflecting on past experiences, what conceptions do children (grades preK-1, ages 4-6 years old) seem to have about the duration of those experiences, as indicated by their descriptions of duration?

In this chapter, I first describe the children who participated in my study. Then, I articulate my qualitative research methodology, drawing from phenomenology (Creswell, 2013; Hycner, 1999) and clinical interview case study (Clement, 2000; Yin, 2003). I then explain how I collected the data. Finally, I explain my use of Wolcott's (1994) *Description, Analysis, and Interpretation* constructs for data analysis with excerpts from my data to demonstrate each stage of analysis.

Participants

I began my study in Spring 2020. Due to Covid-19, and the unanticipated effects it had on children and their families, I had to be thoughtful about my participant recruitment and data collection methodology. I used a stratified, purposeful sample to access children who could help address my research question. I began by reaching out to parents I knew with children between the ages of 4 to 6 from a large metropolitan area in the Midwest, United States. Several of these parents reached out to others they knew with children of a similar age. This stratification allowed me to purposefully assemble a sample of participants from across multiple cities in an effort to provide for maximum variation in the data (Creswell, 2013), as shown in Table 1.

Each participant belonged to a public-school district (District A, B, C, and D) that follows Common Core standards in their school mathematics instruction. Selecting children from preschool, kindergarten, and early first grade allowed me to capture the children's conceptions before school-based exposure to time—as such formal time instruction is first introduced through Common Core in first grade (CCSSI: Measurement and Data, 2020).

Table 1

Participant Demographics

Participant's Name (Pseudonym)	Gender	Age	Grade	Other Information
Kyla	F	4	Preschool	• Sister of Easton • Part of school district D
Cody	M	4	Preschool	• Has younger brother (age 3) • Part of school district C
Mick	F	5	Kindergarten	• Has younger brother (age 2) • Part of school district A
Tanner	M	5	Kindergarten	• Only child • Part of school district B
Easton	F	6	First	• Sister of Kyla • Part of school district D
Shelby	F	6	First	• Only child • Part of school district B
Lennon	M	6	First	• Has older sister (age 9) • Part of school district A

Additionally, given the inherent cultural influences of how children interact with time, as described in Chapters I and II, it is important to note that all participants were from affluent, middle-class homes. Situating the data in such a manner is essential in explicating the exploratory nature of this study. Specifically, the findings can serve as a starting point for future research on children's durational reasoning but cannot be generalized across a broader population.

Qualitative Research Methodology

To provide my participants the opportunity to reflect on their experiences and allow me to infer into their conceptions of duration, I utilized components of the clinical interview case study (Clement, 2000; Yin, 2003) and phenomenology methodologies (Creswell, 2013; Hycner, 1999). By presenting cases of specific children in this study, I intended to establish some

foundational benchmarks of possible conceptions children might have about duration as an attribute of their lived experiences prior to school-based time exposure. My intent was not to provide exemplars for such reasoning, but rather to "map the terrain" of such conceptions. Thus, drawing on phenomenology seemed appropriate.

Phenomenology

Phenomenology is intended to understand the lived experiences of individuals, and the meaning of those experiences for the individuals (Creswell, 2013; Hycner, 1999). During this process, "investigators set aside their experiences, as much as possible, to take a fresh perspective toward the phenomenon under examination" (Creswell, 2013, p. 80). For this study, I strived to set aside my own conceptions about time to understand how children might conceive about the duration of their lived experiences. For example, when I first considered Matti's "ABCs" durational unit from Chapter I, I assimilated his reasoning into my own frame of reference, believing that he had considered the duration of his experiences, not the activity. Through conversations with my advisors, however, I realized that I based my assumptions of his conception on *my* durational reasoning, not his. Situating my own temporal conceptions was an important step in my research process.

Traditionally, phenomenology employs open-ended interviews to gain insight into the participants' experiences. This methodology is quite different from other studies of children's conceptions of time (Earnest, 2017; Kamii & Russell, 2010, 2012; Levin, 1977, 1979; Levin & Gilat, 1983; Levin et al., 1978; Long & Kamii, 2001; Piaget, 1969; Russell, 2008). In those studies, the researcher led participants through various time-based experiments. For example, Piaget (1969), Long and Kamii (2001), and Russell (2008) had children use water flowing through different vessels to serve as non-standard timers for activities. Earnest (2017) had

children use different models of analog clocks to reason about duration. Each of these studies used the in-the-moment, physical durational experiences on which the children reflected.

Instead, drawing from a phenomenological methodology allowed me to explore the participants' conceptions as they led me (the researcher) through their experiences and understandings. Rather than providing the tools of time (e.g., clocks, hours, minutes; Earnest, 2017, 2019), or incorporating other time-related constructs (e.g., speed or length; Kamii & Russell, 2010; Long & Kamii, 2001; Piaget, 1969; Russell, 2008), I attempted to formulate "an initial description of the subject's mental structures, goals, and processes that provides an explanation for the behavior exhibited" (Clement, 2000, p. 575) through exploratory clinical interviews.

Clinical Interview Case Study

Case study can be valuable when "the boundaries are not clear between the phenomenon and the context" (Yin, as cited by Baxter & Jack, 2008, p. 545) in which they occur. Because time is inseparable from the context in which one's mental system is re-presenting (reconstructing) it, case study methodology provided me the opportunity to explore each participant's durational conceptions. From these I could then infer the boundaries of the duration of their experiences within their unique context. To begin mapping my participants' conceptions, I drew on Clement's (2000) exploratory clinical interviews.

Clement (2000) detailed modifications that have been made to clinical interviews since Piaget used this methodology in his foundational studies. To this end, he elaborated on Piaget's clinical interview as a way for systematically observing children in action as a "means of accessing a child's inner reality" (Mayer, 2005, p. 371). This objective aligns with a phenomenological methodology.

Piaget's traditional clinical method was a three-part process, beginning with the child's explanation of the phenomena under investigation prior to the experiment. The child then observed or completed a task, verbalizing their thinking as they proceed. Finally, the child explained what happened, as they understood it.

Clinical interviews have since evolved, allowing for more open-ended techniques to gather information around participants' conceptual understandings (Clement, 2000). Clement (2000) explained that in "some exploratory varieties of clinical interviewing, the investigator can also react responsively to data as they are collected by asking new questions in order to clarify and extend the investigation" (p. 547). Considering this stance as consistent with the phenomenology perspective, I followed this exploratory framework during my semi-structured interviews.

Interview Questions

During my first interview, I followed a semi-structured interview protocol consisting of a set of questions to guide the overall structure of each initial interview, as shown in Table 2. I asked these questions to promote my participants' reflections on and expressions of the duration of their lived experiences. Particularly, I attempted to phrase the questions in a general way to provide space for reflection, while also increasing the likelihood of a child referring to both the boundaries (start/stop) and in-between aspects of their thinking about duration.

I began by asking each participant some general "get to know you" type questions. I intended these to put the young participants at ease with the format of the interview in a virtual environment. They were also meant to provide possible prompts of familiar experiences, unique to each individual child, if needed. From these, I turned to focused questions about the duration of different experiences.

Table 2

Interview Protocol

Question #	Initial Question	Follow-Up Questions		
"Get to know you"	What are some activities you enjoy doing?	What do you like to do on the weekend?	What is your favorite part of school?	What is something you don't like to do?
1	What is something that takes a long time?	How do you know it takes a long time to do?	Can you think of something that takes longer?	How does this compare to the other activity?
2	What is something that takes a short time?	How do you know it takes a short time to do?	Can you think of another activity that takes the same amount of time?	How do you know these two activities take the same amount of time?
3	Tillman and Barner (2015) linguistic task with modified events: Provide card with words and pictures of familiar events (watching a movie, brushing your teeth, sleeping at night, eating your lunch)	Organize these events from shortest duration to longest duration	Why did you order these events this way?	Discuss any relationships between the length of events that appear
4	How do you know how long things take to do?	How can you measure time?	What if there were no clocks/watches/timers /etc.?	How would this tell you how long something takes to do?
5	Berggren (2018): What is time?			

Reisman (1971) explained that smaller units of time are more difficult for children to conceptualize. I was curious if I might be able to infer similar (or different) reasoning by asking questions about experience of longer duration (Question 1) and shorter duration (Question 2). I then used a modified task from Tillman and Barner (2015) to see how each child compared and

organized the duration of familiar events (Question 3) I chose this task because it is theoretically different than those used by Piaget (1969); rather than an in-the-moment experiment, Tillman and Barner's (2015) task allowed the participants to reflect on past experiences as a means of accessing their reasoning. Building from this durational comparison, I asked about quantifying durations to measure time to elicit examples of boundaries in my participants' experiences (Question 4). I presented this question after the previous three to avoid orienting the participants toward the quantification of time before allowing them to reflect on the duration of their experiences. I intended the final question, "What is time?" to explore how each child conceived of time as a thing.

After each pre-set question, I asked follow-up, unscripted questions to elicit each child's descriptions and thus increase my ability to infer into their conceptions. These follow-up questions depended on the participant's individual response. For example, the final question I asked all participants was "What is time?" Depending on how the child answered, the follow-up questions varied. This can be seen in an unscripted, follow-up question I asked of 5-year-old Mick, in Excerpt 3.1.

Excerpt 3.1 (August 2020): Example of Mick's Response to My Follow-up Questions

1. A (Author): What is time?
2. M (Mick): [looks around and hesitates for 5 seconds][4]
3. A: Have you ever heard that word before?
4. M: No.
5. A: Mom's never said it's time to go?
6. M: No.

[4] I used [brackets] to show the participant's actions and hesitations.

Mick's hesitation (Line 2) signaled me to re-word the question in a closed—yes or no—manner, "Have you ever heard the word before?" (Line 3). It is worth noting that this question came after 15 minutes of questions such as, "What is something that takes a long time?" (Question 1) and "How can you measure time?" (Question 4), where both Mick and I had used the word "time." My follow-up question, "Mom's never said it's time to go?" (Line 5) was meant to prompt a specific, common example of the word "time" in a context that may be familiar to Mick. Her response indicated that even in context, Mick seemed either unwilling or unable to conceptualize time as an attribute of her experiences.

Another participant, Shelby (age 6), responded differently to the same question, thus my follow-up questions were also quite different, as shown in Excerpt 3.2.

Excerpt 3.2 (August 2020): Example of Shelby's Response to My Follow-up Questions

1. A: What is time?
2. S (Shelby): [hesitates briefly] Um, time is like, um [hesitates and makes a clicking noise] um, if you don't have any other things and all that stuff, um, time is [hesitates briefly], yeah, I don't know [laughs]
3. A: Have you ever heard your mom or dad talk about time before?
4. S: Yeah, [nods] like about numbers and their phones.
5. A: Okay, so what do they say about time?
6. S: [hesitates briefly] It's late.
7. A: What does that mean if they say it's late?
8. S: That means you **have to**[5] go to bed.

[5] I used b**old text** to show the participant's emphasis during their responses.

I intended the follow-up question, “Have you ever heard your mom or dad talk about time before?” (Line 3) to figure out if Shelby’s response “…yeah, I don’t know” (Line 2) was a lack of understanding of the term “time” or if it was a difficulty in verbalizing what she had re-presented. Shelby’s response to my follow-up followed a line of reasoning from numbers on her parents’ phones (Line 4) to it being late (Line 6), to going to bed (Line 8). This allowed me to infer that to Shelby, time seemed to be an attribute she related to her experiences.

I arranged follow-up interviews with each child about a month after their initial interview. The format for the follow-up interviews stemmed from each individual child’s responses during their initial interview. Immediately following each initial interview, I made notes on questions I still had about how the child conceived of duration as a measurable attribute of their experiences. These questions became the starting point for the follow-up interviews, which were somewhat unstructured conversations.

To begin the follow-up interviews, I asked each child what they had been up to since our last conversation. This provided insight into some additional experiences that we might discuss. From this, I addressed any questions I had from the first round of interviews. For example, during the first interview, 5-year-old Tanner had explained that painting his house took a long time to do. When asked how he knew it took a long time, he responded, “Because it does.” During the follow-up interview, I again asked Tanner to reflect on painting his house, as shown in Excerpt 3.3.

Excerpt 3.3 (September 2020): Example of Follow-up Interview Structure from Tanner’s Description of Painting his House

1. A: How do you know it takes a long time to paint your house?

2. T (Tanner): Well, first of all our house was brown and but then we painted it to be **green**.

3. A: Oh, I love green! How did you know it took a long time to paint your brown house green?

4. T: [hesitates briefly] Because it took **the longest time ever**, like for 40 minutes.

5. A: And what would you do to paint the house?

6. T: Use the paintbrush and you would paint the bottom then use a roller to paint the top.

7. A: Oh, so you have to use different things to paint it with?

8. T: Yep.

9. A: Does it paint faster with the paint brush or the roller?

10. T: Roller's faster and for the tippy tip of the chimney you use fingerprints.

11. A: Oh, really? You get to do some finger painting, too!

12. T: Yup.

13. A: I didn't know that! How fun! So, when you're doing the roller you said the roller is faster. Does that make it take less time?

14. T: Um, less time.

15. A: How come?

16. T: Because of how **long the day is**.

When asked to elaborate on his experience of painting a house, Tanner was able to describe specific durational concepts, specifically, "…it took the longest time ever, like for 40 minutes." (Line 4), "Roller's faster…less time." (Line 10, 14), and "Because of how long the day is" (Line 16). Tanner's initial interview gave little evidence of his consideration of durational attributes of painting. However, when asked to elaborate on his experience of painting his house, Tanner gave numerous indications of his reflection on duration as an attribute of his experiences.

Data Collection

In clinical exploratory studies, Clement (2000) explained, "sensitivity to subtle observations is important" (p. 575). Therefore, I video recorded each interview to capture nuances in words and actions of the participants for future review and analysis. In math education research, it has been found that gestures can "help make inferences about the quantitative operation" (Hornbein, 2015, p. iv) being used by the participant. Hornbein (2015) described beat and deictic gestures as providing emphasis and identifying space, respectively. For example, in this study a child who very slowly bops their head and drags out their words as they describe an experience that feels long, may be emphasizing this perception through their gestures.

Video recording allowed me to capture nuanced gestures such as those noted; however, I also used an additional audio recording in case of technical, video difficulties. I considered this assurance necessary because I did not take written observational notes during the actual interview. I did this for two reasons, 1) to provide complete focus on the participant's reasoning so that I could provide purposeful follow-up questions, and 2) to minimize distracting the participants during their reflection (Saldana & Omasta, 2018).

Immediately following each interview, I completed reflective remarks on ideas that stuck out as interesting or pertinent (Miles & Huberman, 1994). During data collection, those remarks provided additional information to analyze while starting to articulate possible themes within and across the data. Taking those notes also allowed me to focus more effort during the interview while informing later, retrospective analysis. From my previous example of Tanner, I found while conducting my initial interview that many of his comparisons were durationally accurate (e.g., it takes longer for an apple tree to grow than it does to paint a house), however, while in the

interview it seemed that the only real reason that he gave for his reasoning was “Because it is.” I made note of this observed trend after the interview, so I could make an explicit effort to look for counter-indicators during my transcription and follow-up interview.

I transcribed each interview with specific attention to the participant’s: (a) vocal inflections (e.g., voice moving up like a question (denoted by (?)) or **emphasized words** (denoted by **bold font**); (b) pauses (e.g., hesitation in their response to the initial question, or pauses during answers (denoted by [bracketing] within the transcript)); and (c) actions/gestures (e.g., looking up while answering, or stretching their arms out to show length (denoted by [bracketing] within the transcript)). In line with my exploratory clinical interview framework (Clement, 2000), attending to a broader spectrum of observations allowed for more “insightful hypotheses that would otherwise be difficult to attain” (p. 575). To guide my analysis toward such hypotheses, I employed Wolcott’s (1994) three-part framework of *Description, Analysis, and Interpretation* in conducting qualitative analysis.

Data Analysis

In attempting to understand how children might conceive of duration as an attribute of their lived experiences, I needed to infer their conceptions. With the exploratory nature of this study, a structured analysis methodology allowed me to strengthen the credibility of my findings. To this end, I employed Wolcott’s (1994) three-part analysis framework, *Description, Analysis, and Interpretation*, which is meant to create “data out of experience” (p. 13). This framework allowed for rich accounts of the participants’ experiences, while providing specific, detailed examination of the experience through a constructivist lens. Below, I expound on each of those three parts.

Description

To begin, I reread my reflexive notes and accompanying transcripts to narrow my data set to relevant passages. Using Wolcott's (1994) *description*, I incorporated all words, actions, hesitations, inflections, and notes from each participant to "address the question, 'What is going on here'?" (p. 12). This allowed me to highlight specific excerpts in detail. This process provided me with the opportunity to let the participant "tell their own story," by presenting their exact words (and my description of their actions), as shown in Excerpts 3.1 and 3.2 of Mick and Shelby's responses to my follow-up questions. However, there is always an element of researcher bias and unintended influence (e.g., the questions being asked, or the actions being acknowledged). Thus, Creswell (2013) emphasized an important step in phenomenology is for the researcher to situate themselves in the phenomenon.

As illustrated through the epistemological review in Chapter I, time is inextricably linked to perception. As an adult, I understand that my actions and emotions have no impact on the duration of my experiences. While it may feel like "time drags on" when I am bored or inactive, or that "time flies" when I am enjoying myself or learning a new activity, this is just my perception and might not reflect the actual passage of time. From a constructivist lens, I also recognize that my understanding of time is not the same as other people's understandings and I strive to create a second-order model of children's possible conceptions (Hodkowski, 2018; Steffe, 1995). For example, when Tanner explained that painting takes a long time, I immediately reflected on painting my own house and the amount of time it took. During my interview, however, I assumed ignorance toward this concept to understand Tanner's conception. This allowed me to listen to Tanner's story and reflect on his reasoning, as I encouraged him to describe his experience of this long time.

Analysis

After identifying the description of the data, Wolcott (1994) explained *analysis* as explaining, "how things work" (p. 12). The process of analyzing data began with what descriptions I chose from the data set. Deciding what excerpts from the interviews best highlighted the phenomenon was an analytic choice. I initially focused on descriptions that may have indicated my participants' internalization of duration as an attribute of their experience (as described in Chapter II; von Glasersfeld, 1995). This selection was an important factor in my final presentation of the data, as it narrowed my participants' descriptions to those instances where I might be able to infer into their conceptions.

In Excerpt 2.2, for example, Reece reflected on the duration of brushing her teeth as "five minutes." As she was unable or unwilling to elucidate her reasoning, I could not infer into her conception of the duration of her experience. I therefore characterized Reece's conception as an example of duration through sensorimotor actions, where duration existed within the experience itself. And while there may be powerful information to be gained from such sensorimotor durational perceptions, it was not the primary focus of this study. Rather, I chose to explore the durational conceptions that I could infer from my participants' descriptions. For example, Excerpt 3.4 continues Shelby's response during Excerpt 3.2.

Excerpt 3.4 (August 2020): Shelby's Continued Response to My Follow-up Questions

1. A: What does that mean if they say it's late?
2. S (Shelby): That means you **have to** go to bed.
3. A: You have to go to bed [laughs] so is that part of time?
4. S: Yeah.
5. A: How is that part of time?

6. S: Because [hesitates as she makes a clicking noise] time is like when in outer, in outer space [puts arm out] there's like, so **the Sun** and **the Earth** spin around, and [hesitates briefly] and we don't feel on Earth and, um, it does time too.

7. A: Whoa! How does that work? How is that time?

8. S: Mmm [hesitates briefly] um [hesitates for 4 seconds as she looks around]—I don't know.

The beginning of Shelby's explanation demonstrated an apparent interiorization that time is an attribute of her experiences ("That means you have to go to bed" Line 2). When asked to explain how having to go to bed was part of time, Shelby's reasoning changed from a personal, re-presented account to a more abstract, scientific conception of the Earth's rotation and revolution. When asked to reflect on this changed understanding, Shelby was hesitant (demonstrated through her pauses, vocalization, and actions), before stating, "I don't know" (Line 8). Here, it seems that Shelby has not interiorized how the rotation of the Earth correlates to the passing of time.

By leaving this continuing excerpt as part of the data, it seems unclear how Shelby conceives of time—whether she conceives of it as part of her experiences or "out there," separate from herself. I would infer, however, that her explanation of the rotation of the Earth is more an indication of an accepted idea from her parents or others in their social milieu (Piaget, 1969), not necessarily her own conceptions. This example illustrates how descriptive and analytic decisions can impact the interpretive findings.

Two other methods Wolcott (1994) explained for analysis were (a) to create displays for the descriptions or (b) to compare one case with another. Displays can allow for visual presentation of the data as a way to validate established conclusions and open the research to further areas of investigation (Miles & Huberman, 1994). Spradley (1979), for example, outlined

various qualitative analysis displays, including a taxonomic analysis. Taxonomic analyses can illustrate the depth of a participant's cultural meaning of a concept. For example, I asked 6-year-old Lennon how he could measure how long something took him to do. Over the course of our conversation, he explained the measurement three different ways (with a clock, think in your head, with both your head and a clock, I don't know). He elaborated on each way at different levels—as I demonstrated by the deepening color in Figure 2. This display allowed me to compile Lennon's conceptions into a single form that held close to his descriptive account, a key aspect of Wolcott's (1994) analysis.

Figure 2

Taxonomic Analysis (Spradley, 1979) of Lennon's Explanation of Measuring Time

<table>
<tr><td rowspan="8">You can "probably" measure time (by)</td><td colspan="4">"I don't know"</td></tr>
<tr><td>"Looking at a clock"</td><td colspan="3">"You can see what time it is"</td></tr>
<tr><td rowspan="2">"You just think in your head"</td><td colspan="3">"If you're really good at math and stuff then you could probably figure it out in your brain"</td></tr>
<tr><td colspan="3">"About time"</td></tr>
<tr><td rowspan="4">"Think in your brain or just look at the clock"</td><td rowspan="3">"It could actually tell you because if it passed a minute then it will probably be, if it's like 1:20 it would be 1:21"</td><td>"A minute is kind of like a second but longer"</td><td colspan="2">"Um," a minute is "probably like 20 minutes" longer than a second</td></tr>
<tr><td rowspan="2">"Uh [hesitated] an hour" is longer than a minute</td><td rowspan="2">"Uh," an hour is "a thing that there's multiple minutes"</td><td>An hour is "probably like 50 minutes? 60?!"</td></tr>
<tr><td>An hour is "Either 50 or 60"</td></tr>
<tr><td colspan="3">"It depends on how, how long it passed"</td></tr>
</table>

Wolcott (1994) also described the use of comparison across cases as a means of inferring similarities among participants' description of the duration of their experiences. When explaining

the phenomenological interview process, Hycner (1985) contended that identifying more generalized themes across interviews, or finding individual variance, are important analytic techniques. As I clustered extracted data by participant and compared across participants, similar themes arose. For example, I inferred that when Tanner explained that it took "the longest time ever, like 40 minutes" to paint his house, he was naming standard units of time that he has heard but might not have a true conception of. I considered this akin to Lennon's description that an hour was either 50 or 60 minutes (Figure 2). This connection across cases may, for example, allow me to highlight common durational and temporal experiences.

As I reviewed the data, I noticed several common conceptions across participant descriptions. I coded these conceptions across cases. Some of these codes included reflecting on duration through speed, attention, size, magnitude, age, length/distance, and activities. During my analysis, I categorized the conceptions under umbrella themes, such as descriptions of magnitude, size, and distance all reflecting duration as a consideration of a gross quantity (described in Chapter II; Piaget, 1965).

By employing varied analysis techniques (creating displays and cross-case analysis), I attempted to create a more comprehensive and credible interpretation of the participants' conceptions of duration as an attribute of their experiences.

Interpretation

Interpretation, Wolcott (1994) explained, consists of the claims made by the researcher. Whereas description and analysis hold close to what the participant presents, interpretation is where "the researcher transcends factual data and cautious analyses and begins to probe into what is to be made of them" (Wolcott, 1994, p. 36). Wolcott cautioned that when reporting on qualitative data, it is better to give too much description and too little interpretation, a notion

supported in phenomenological methodology (Hycner, 1985). As a researcher, I worked during the interpretive process to hold myself in check against the descriptive data.

Wolcott (1994) explained several different approaches to interpretation, including connecting to the personal experience of the researcher or explaining what might still be missing from the data to make a claim about the phenomenon under study. While I do see both as important considerations for future study (see Chapter V), I prefer Wolcott's (1994) interpretive approach of turn to theory in order to link to other studies and larger issues.

As explained in Chapters I and II, I found no existing research specifically looking at children's conceptions of time as an attribute of their experience based on their reflections of past activities. During my interpretation, I deferred to (a) Piaget's (1969) research on children's conceptions of physical time, as he articulated a clear link between conceptions of inner duration and physical time experiences and (b) von Glasersfeld's (1981) perceptual framing within his attentional model as a unitizing operation (see Chapter I and II). Excerpt 3.5 is an example of 5-year-old Mick's description of the duration of drawing a duck versus drawing a flamingo.

*Excerpt 3.5 (*September *2020): Mick's Reflection on how the Rapidity of her Actions Impacts the Duration of her Experience*

1. A: You said on the flamingo that the feathers made it take a long time. How come the feathers on the duck didn't make it take a long time?
2. M (Mick): Because I drawed [sic] it (the duck) **faster**.
3. A: Oh, so because you moved faster it was a shorter time?
4. M: Yeah!
5. A: So, if you drew it slower what would happen to the time?
6. M: It would be longer.

Based on her response, it seems to me that Mick conceived of an inverse relationship between the rapidity of her actions and the duration of her experience. Specifically, it seems that Mick understands that by her actions moving at a quicker pace, she can make the duration of her experience shorter. Piaget (1969, p. 41) noted the inverse relationship ("more rapid=less time") through his durational studies. This conception may be an indication of the transition from pre-operational durational reasoning to what Piaget (1969) termed "articulated intuition of duration" where children begin to "appreciate that time and velocity are inversely proportional" (p. 44). However, at what level Mick might conceive of this relationship (Chapter II, participatory vs. anticipatory; Tzur & Simon, 2004) cannot be inferred from this excerpt. Wolcott (1994) noted that guarding against overzealous, unfounded claims is an important consideration during interpretation.

As mentioned, the purpose of this research was not to identify exemplars or make grand claims about the durational reasoning of young children. Rather it was to mark (Tzur, 2019) possible conceptions children might have about duration as an attribute of their experiences. My interpretations, therefore, cannot be assumed for the general population. Correlations to established theory might be notable but should not be accepted as support for these theories. Rather, the analyses completed (Chapter IV), and the interpretations made (Chapter V) might offer a foundation for future studies on children's conceptions as an attribute of their experiences.

CHAPTER IV

ANALYSIS

Author: Why do you think brushing your teeth takes a short amount of time?

Tanner (age 5): Because I do. I just know it.

In this chapter, I present findings from exploratory clinical interviews with seven children between the ages 4-6. These findings focus on how each child re-presented, or mentally reconstructed, their lived experiences to reflect on duration as an attribute of those experiences, and thus address the following research question:

> When reflecting on past experiences, what conceptions do children (age 4-6 years old) seem to have about the duration of those experiences, as indicated by their descriptions of duration?

The data I present come from two interviews with each participating child. The initial interview was semi-structured and followed a series of questions that drew upon past studies on children's temporal and durational reasoning, as I described in Chapter III (Berggren, 2018; Piaget, 1969; Reisman, 1971; Tillman & Barner, 2015). Then, I conducted a follow-up interview with each child in an unstructured, open-ended, conversational approach. Combined, the initial and follow-up interviews provided each participant the opportunity to lead me (the researcher) through their re-presentations of past experiences, giving me an indication of how they conceived of the duration as an attribute of those experiences. As I explained in the conceptual framework (Chapter II), my analysis focused on what attributes each participant reflected on as they bounded their experiences (von Glasersfeld, 1981) to conceive of the duration as a measurable attribute of those experiences.

As I discussed in Chapters I and II, how people might conceive of duration as a measurable attribute is linked to their conception of quantity (Thompson, 1994). As I will demonstrate, what quality of their experience the participants conceived of as measurable differed greatly. To identify these differences, I focused on each child's re-presentation (von Glasersfeld, 1991) of their experience, that is, the entire sequence of events that the child expressed (and thus recalled and re-ran) as part of the duration of a lived experience.

From data I collected of each child's utterances and actions when asked to recall an experience, common themes emerged across participants. The themes reflect my inferences about the children's *interiorization* of duration as a measurable attribute based on their re-presentations of their experiences. The nature of those themes is consistent with Tzur's (2019) notion of markers. That is, they capture various ways in which children might conceive of duration as a measurable attribute of their experience—not on how those conceptions change over time. Accordingly, I do not intend the themes produced to serves as exemplars for durational measurement nor a progression of durational reasoning.

I organized this chapter around three main themes: (a) duration as an accumulation of activities completed (Piaget, 1969; Steffe, 2013); (b) duration as a consideration of a gross quantity (Piaget, 1965); and (c) duration as a result of exertion (Piaget, 1969). I will articulate the meaning of each theme in its respective subsection.

Before presenting the themes, I illustrate my data analysis through a conversation I had with one, 6-year-old participant (Shelby). I chose this excerpt because it provides a glimpse into all three themes. First, I *describe* (Wolcott, 1994) my conversation with Shelby. Then, I demonstrate key elements of my *analysis* and *interpretation* (Wolcott, 1994) of the three themes being presented.

Illustrating Data Analysis through a Conversation with Shelby

As Shelby's computer connected to Zoom for our follow-up interview, I could only see the top of her ponytail. She was sitting on the floor of her parents' home office, surrounded by piles of colorful cards. When she saw me, she excitedly jumped up and started showing her new collection of Pokémon cards (see a sample of Pokémon cards in Figure 3). Shelby explained that she got some new cards the previous day as a reward from her parents, and she had been organizing them into binders ever since. Without being prompted, she mentioned that it was taking her a long time. I thus followed by asking her how she knew it was taking a long time. Shelby stated that because she had a lot of cards, and it was hard to put the cards in the binders, it was taking a long time.

Figure 3

Sample Pokémon Card Collection

Shelby's explanation piqued my interest in her focus on the effort, or exertion, of her actions (Piaget, 1969) and how she might reflect on this as she considered the duration of other

experiences. I followed by asking: Could [you] think of another activity that felt "hard?" Shelby responded, "vacuuming." Interestingly, Shelby had discussed doing household chores during her first interview when asked about things she did not like doing. I invited her to tell me more about vacuuming (Excerpt 4.1).

Excerpt 4.1 (September 2020): Shelby's Description of Vacuuming

1. A (Author): Can you think of something else that's hard for you to do?
2. S (Shelby): Probably vacuuming!
3. A: Tell me about vacuuming.
4. S: Um, because, um, this vacuum's like, um like, uh **that tall** [puts arms up to show the height][6] and I can't really, like; I can reach it, but it makes, like, a lit—it makes, like, **a long time** because I have to push it and **it's heavy**.
5. A: If the vacuum were lighter, would it still take a long time?
6. S: No, because if it was lighter, I could just push and push.
7. A: So, you think the reason it takes a long time is because it's so heavy.
8. S: Yeah.
9. A: So, when your mom vacuums, do you think it takes just as long as when you vacuum?
10. S: No. She's the amount of weight of the vacuum.
11. A: Oh, okay, so because your mom weighs the same as the vacuum, it's shorter for her?
12. S: [nods and laughs]
13. A: [laughs] So, you said vacuuming was hard for you. Does that make it take a long time?
14. S: Uh, pro— (I infer she was starting to say probably, then changed to) Yeah.
15. A: Yeah? What if you only vacuumed one room?

[6] As I described in Chapter III, I used brackets within interview excerpts to show the participants' [physical actions or hesitations], and bold font to highlight **emphasis verbalized** by the participants during their responses.

16. S: That would probably take, like, a minute.

17. A: Is a minute short or long?

18. S: Short.

19. A: Why would it be short?

20. S: Umm, because [hesitates and makes a clicking noise] hmm [hesitates briefly] umm, probably takes, like, a minute because it's, like, one room, but **first** you have to, like, put stuff in your bed, not under. So, you vacuum **everywhere**.

21. A: Is that part of vacuuming, having to clean up the floor and everything first?

22. S: Yeah.

23. A: Does that make it take longer?

24. S: Uh, **yeah** [signs heavily] yeah, it takes, like, a long, **long, long** time.

25. A: Is there any room in your house that maybe doesn't take as long to vacuum?

26. S: Bathroom.

27. A: How come?

28. S: Well [laughs] well because, it's, like [puts arms out about shoulder width apart] this big.

29. A: Okay, how does that compare to the other rooms?

30. S: Well, we have, like, it takes, like, like, the other rooms are, like, **big**.

31. A: So, the other rooms are big, and you said the bathroom is, like, this [spreads arms similar to what Shelby had], so you're saying the bathroom is smaller?

32. S: Yeah, and **especially** mom's room, she has a bathroom inside her room and the bed, like [spreads arms shoulder width] this big and then their whole room's, like, this [spreads arms as wide as she could].

33. A: Okay, and what if you had to vacuum your mom's room, how would you feel about that?

34. S: Hmm, a hard time.

35. A: How come?

36. S: [makes clicking noise] Well my, my room is, like, like [spreads arms a bit wider than shoulder width]

37. A: Okay, and your mom's room is, like—

38. S: [spreads arms as wide as she could]

39. A: Okay, so which room would take longer to vacuum?

40. S: Mom's.

41. A: How come, just because it's bigger?

42. S: [nods affirmingly]

As Shelby described her re-presented experiences of vacuuming, she seemed to conceive of different attributes of her experiences to quantify the duration. For example, she seemed to coordinate duration relative to the height and weight of the vacuum (Line 4), to the size of the rooms being vacuumed (Line 15-42), and to various activities involved in vacuuming (Line 20-22). To classify how Shelby conceived of the duration of her experience, I created a thematic analysis (Nowell et al., 2017) of the three themes identified, shown in Figure 4. This display illustrates Shelby's conceptions of duration as an accumulation of activities completed (color coded green), a consideration of gross quantities (color coded blue), and a result of exertion (color coded red). Additionally, the analysis in Figure 4 illustrates how Shelby's reasoning was occasionally interdependent across the three themes. For example, when Shelby explained the potential activity of vacuuming her mom's room, she described the size of the room (Line 32; gross quantity) along with her view that it would be "a hard time" (Line 34; result of exertion) to explain that it would take a longer time than vacuuming her own room.

Figure 4

Thematic Analysis (Nowell et al., 2017) of Shelby's Vacuuming Description

Theme Coded		Shelby's Description	Inferred Duration
The duration of vacuuming as…	An accumulation of activities completed	"push it" (Line 4)	"takes a long time" (Line 4)
		"push and push" (Line 6)	When considered with "lighter" vacuum (consideration of a gross quantity) and "just" (result of exertion) would take a shorter time (Line 5-6)
		"first you have to put stuff in your bed" (Line 20)	"takes, like, a long, long long time" (Line 24)
		"vacuum everywhere" (Line 20)	
	A consideration of a gross quantity	"tall" (Line 4)	"takes a long time" (Line 4)
		"heavy" (Line 4)	
		"lighter" vacuum (Line 6)	When considered with "just" (result of exertion) and "push and push" (accumulation of activities completed) would take a shorter time (Line 5-6)
		"mom is same weight as vacuum" (Line 10)	Shorter for mom (Line 11-12)
		Vacuuming one room = "a minute" (Line 15-16)	"a minute is short" (Line 17-18)
		"a minute because one room" (Line 26)	
		Bathroom is "like [puts arms out about shoulder width apart] this big" (Line 28)	Doesn't take as long (Line 25-26)
		"other rooms are, like, big" (Line 30)	
		"bathroom is smaller" (Line 31)	
		Mom's room is big, "has a bathroom inside her room… bed, like [spreads arms shoulder width]" (Line 32)	It takes longer to vacuum (Line 52-53) and would make "a hard time" (result of exertion) (Line 34)
		It's bigger (Line 36-38)	
	A result of exertion	"hard" (Line 1-2)	"takes a long time" (Line 4)
		"I have to push it" (Line 4)	
		"just push and push" (Line 6)	When considered with "lighter" vacuum (consideration of a gross quantity) would take a shorter time (Line 5-6)
		"a hard time" (Line 34)	When considered with Mom's room is big, "has a bathroom inside her room… bed, like [spreads arms shoulder width]" (consideration of a gross quantity) (Line 32) takes longer to vacuum (Line 39-40)

To delineate the three themes more explicitly, I highlight each within the context of my conversation with Shelby, beginning with her reflection on duration as an accumulation of activities completed. First, I conjecture that when my participants conceived of *duration as an accumulation of activities completed*, they seemed to attend to the discrete activities that were completed during the overall experience. These activities might be a repeated task (e.g., when Shelby explained you "push and push" (Line 6)). They may also be an accumulation of different activities (e.g., pushing a vacuum (Line 4 and Line 6), cleaning up before vacuuming (Line 20), and vacuuming everywhere (Line 20)).

When conceiving of the duration of vacuuming, Shelby seemed to partition her experience into smaller units of her imagined (interiorized) activity (Steffe, 2013), for example cleaning and then pushing the vacuum everywhere. For Shelby, it seemed obvious that each of these sub-activities must necessarily be shorter than the overall experience of vacuuming, and thus accumulated to quantify the overall duration of her experience.

It is important to note, that, to Shelby, these sub-units of activity were not standardized in any way, and they were not being counted. Shelby did not seem to acknowledge the duration of cleaning as being the same duration as pushing the vacuum, in such a way that two iterations of cleaning might equal one vacuuming experience. Rather, I interpreted her accumulation to indicate non-iterated units of one (Long & Kamii, 2001), where each "one" is its own unique quantity. In other words, Shelby seemed to be reflecting on a non-enumerated accumulation of activities completed. Combined, both aspects of her experience (no standardized units, non-enumerated) indicate to me that Shelby has not yet unitized duration as a conception of activity accumulation (Steffe, 1991; von Glasersfeld & Steffe, 1985).

Secondly, when my participants conceive of *duration as a consideration of a gross quantity*, I infer that they seemed to reason about a perceptual relationship between two attributes of their experience (Piaget, 1965). Through my analysis, I found these attributes focused on either the magnitude of a feature of their experience (Piaget, 1965), the relative size of a specified aspect of the experience (Piaget, 1965), or the distance traveled during that experience (which Shelby did not present in Excerpt 4.1; Piaget, 1969).

Shelby described the duration of vacuuming through multiple aspects of gross quantity, both explicitly and implicitly. She implicitly reflected on the magnitude of rooms when explaining that vacuuming "one room"—compared to all the other rooms in the house—would take 1 minute, which in her mind seemed to be a term associated with a short duration (Line 15-16). When Shelby described *the size* of the vacuum compared to herself or to her mom (Line 4, 6, 10), she seemed engaged in explicit comparison of a gross quantity. I interpreted that this allowed her to quantify the duration of vacuuming in relative terms, namely, longer, and shorter—or intensively, as defined by Russell (2008).

Thirdly, when participants described *their* efforts or the rapidity of *their* actions, I interpreted they conceive of *duration as a result of exertion*. This means that they considered how the exertion imagined to be involved in their intentional actions might impact the duration of their experiences (Piaget, 1969). For example, when Shelby explained that a lighter vacuum would take her less time, she stated, "I could just push and push" (Line 6). I infer Shelby's use of the word "just" indicated that, to Shelby, the experience of vacuuming would be durationally different (shorter) because performing the task would be easier. Shelby's explanation of putting her Pokémon cards into binders seemed to support this inference. Excerpt 4.2 shows Shelby's response when I asked if there was a way that she could make such sorting easier.

Excerpt 4.2 (September 2020): Shelby's Description of her Perceived Effort of Organizing Pokémon Cards

1. A: Is there a way that you could make (organizing) easier?
2. S: [hesitates for 4 seconds] Probably? If you have cards that you didn't put in, you just need to put **those** in **first** then you need to put another one.
3. A: And that would make it easier?
4. S: Yeah.
5. A: Do you think if you made it easier, it would make it take longer or shorter?
6. S: Probably shorter?

To me, it seemed that Shelby conceived of a reciprocal relationship between her efforts and the duration of her experiences, as opposed to focusing on the amount of activities involved in sorting. In Line 2, Shelby's statement, "you just need to put those in first" indicated, to me, her reflection on the ease of the activity, not necessarily the sequence of events completed. I infer that to Shelby, the easier her perceived exertion, the shorter the duration. I interpret this to be similar to the inverse relationship between velocity and duration that older individuals typically construct (i.e., the faster something moves, the shorter the duration; Piaget, 1969).

The conceptions presented through Shelby's illustration demonstrated the three themes that I identified during my analysis. From her description, I infer that Shelby's re-presentation of vacuuming revealed a reflection on the duration of her experience through her accumulating activities, consideration of gross quantities, and her perceived exertion. Through the remainder of this chapter, I present examples of how all seven of my participants internalized their experiences, through their descriptions of the re-presented sequence of events, then interiorized the duration of their experiences as either: (a) an accumulation of activities completed; (b) a

consideration of a gross quantity; or (c) a result of their exertion. I begin with duration as an accumulation of activities completed.

Duration as an Accumulation of Activities Completed

To demonstrate how a child might conceive of duration as an accumulation of activities completed, this section includes excerpts from all seven participants. I selected passages to provide a variety of reflections across two different sub-themes: repeating a single activity multiple times and accumulating different activities in a single experience. For each excerpt, I *describe* (Wolcott, 1994) how the child re-presented activities to explain the duration of their experience. From these descriptions, I inferred how each child displayed the accumulation of activities to explain the duration of their experience. I *analyze* (Wolcott, 1994) how each excerpt demonstrated the participant's conception of duration as an accumulation of activities. Additionally, I compared across cases to note similar reasoning between participants. Finally, I *interpret* (Wolcott, 1994) how the participants' conceptions indicate their unitizing operation of duration as an attribute of their experience (Steffe, 1991; von Glasersfeld, 1981; von Glasersfeld & Steffe, 1985) and how they aligned with past research. I begin by comparing two participants' (Kyla and Mick) descriptions of a repeating activity within a single experience as they reflect on the duration of their experience.

Repeating One Activity Multiple Times

Kyla (preschooler, age 4) and Mick (kindergartener, age 5) each described a particular experience of drawing a picture as they reflected on the long duration of their experience. As each described their experience, I inferred that the "long" duration resulted from the child's re-presentation of a repeating activity. I begin with my conversation with 4-year-old Kyla.

During her initial interview in August 2020, Kyla told me about beginning her time at preschool. She mentioned that she enjoyed math because she got to learn about big numbers "like 10," which she proudly displayed on her fingers. She explained that she did not enjoy sitting at school, "you just get to sit and sit and sit," she sighed. While sitting in preschool, Kyla described tracing different shapes on sandpaper. This, she explained, took less time than when her teacher drew a picture of an animal. I asked Kyla how she knew it was longer for her teacher to draw the picture. Excerpt 4.3 shows her response.

Excerpt 4.3 (September 2020): Kyla's Description of the Duration of her Teacher Repeatedly Drawing

1. K (Kyla): It takes a while for Miss Skylar (teacher, pseudonym) to do it.
2. A: Why do you think it takes a while?
3. K: Because, she, she, sometimes she gets it wrong and then she has to do it **again**.
4. A: Oh, so because she has to do it again, it takes longer?
5. K: Yes.

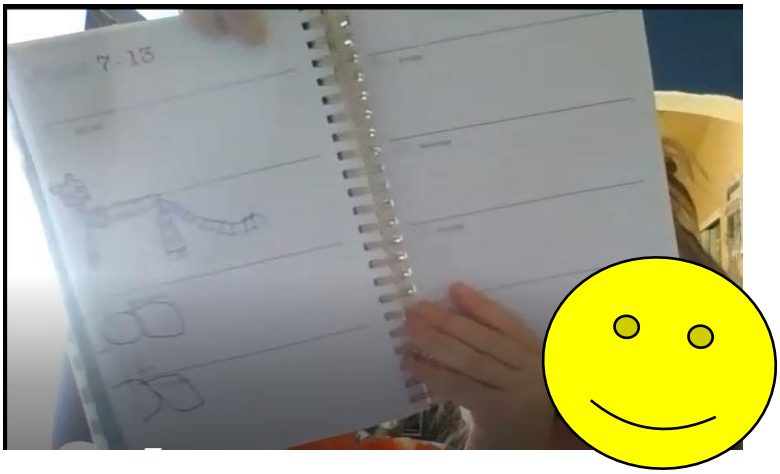
To me, Kyla's emphasis of the word "again" (Line 3) was an important consideration as she conceived of the long duration of her experience of watching her teacher's drawing. I infer that Kyla was clear in her conception that her teacher's drawing took longer because she made a mistake causing her to repeat her activity. Kyla did not seem to consider all school related drawing experiences to be durationally similar. While she described tracing shapes on sandpaper, she indicated that she traced multiple shapes, which she explained took "a little bit…just because it does." She seemed to conceive of a repeated activity as taking longer than non-repeated tasks, as she re-presented her teacher, Miss Skylar's, drawing. I conjecture that Kyla implied the

boundary of this drawing experience (von Glasersfeld, 1981), where Miss Skylar drew and "got it wrong" (Line 3), then started and completed the drawing again within a single experience.

I found Kyla's reflection similar to a conversation that I had with 5-year-old Mick. During our follow-up interview, I noticed that Mick was looking down a lot—all I could see was the top of her head bopping up and down in the camera. I asked what she was looking at, and she responded that she was drawing pictures of different animals. She began drawing after a conversation we had about a recent trip she had taken with her family to the zoo. I asked Mick if she would show me some of the pictures she had drawn, an example shown in Figure 5. As she presented her picture, we talked about what she had drawn, presented in Excerpt 4.4.

Figure 5

Mick's Drawing of a Tiger



Note. Throughout the last part of her follow-up interview, Mick drew pictures of the different animals she had seen on her recent trip to the zoo.

Excerpt 4.4 (September 2020): Mick's Description of the Duration of Drawing a Tiger

1. A: Can I see what you're drawing?
2. M (Mick): I, uh, I started drawing an **elephant**, but I drawed [sic] a tiger **first**.

3. A: Let me see [Mick shows drawing to the camera, Figure 5] oh my goodness, you're a great artist. That is such a good tiger. How long do you think it took you to draw that tiger?

4. M: A **long** time.

5. A: How come?

6. M: Because, because the, I had to do it **all over again** because I had to get this drawing.

Like Kyla, I infer that Mick's emphasis of "all over again" (Line 6) was significant in her consideration of her repeated activities impacting the duration of her experience. It was unclear to me, however, what Mick had to do all over again that might have made the duration of her experience longer. I asked Mick to describe her experience of drawing, so that I could better understand how she was bounding her repeated drawing (von Glasersfeld, 1981):

7. M: [hesitates briefly as she looks at her drawing] Uh, I drawed [sic] **the head** and the, and the [hesitates briefly] and the ears.

8. A: And then what?

9. M: And then, uh, and then I started drawing the legs **and the tummy**.

10. A: And then what?

11. M: And then the tail.

12. A: And then?

13. M: [hesitates for 4 seconds as she looks at her drawing]

14. A: How did you know you were done?

15. M: Because I—and then I started doing the stripes and **then** I was done.

16. A: So, you finished when you got done with the stripes?

17. M: Yeah.

Mick's description of drawing the tiger did not seem to include a repetitive process. Rather, to me, Mick clearly explained each unique step of a single drawing. She bound her experience (von Glasersfeld, 1981) from the start of her drawing, the head, and ears (Line 7) to the conclusion, the stripes (Line 15-17). It seemed, therefore, that Mick may have conceived of the long duration of her experience based on an accumulation of activities (each step of her drawing) rather than her initial claim that she had to do it all over again (Line 6). Perhaps, she had drawn another picture of a tiger on a different page (Figure 5), which she re-presented in her initial response but did not account for in her final description.

Accumulating Different Activities

Tanner, Cody, and Shelby seemed to re-present the duration of their experiences through an accumulation of several different activities, like Mick's description in the continuation of Excerpt 4.4. During his initial interview, I asked 5-year-old Tanner to organize four common experiences from the shortest duration to the longest duration (brushing his teeth, sleeping at night, eating lunch, watching a movie). This interview question built from Tillman and Barner's (2015) linguistic study (described in Chapter II) by encouraging each participant to explain how they conceived of the durations of the four experiences.

As Tanner explained his organization of the four experiences, he stated specific, standard-unit durations for each experience. According to Tanner, brushing his teeth took 5 minutes, sleeping at night took 6 minutes, and eating lunch took 15 minutes. Tanner's measurements seemed similar to Reece's (Chapter II) use of standard units, which may or may not have reflected the actual elapsed time of his experiences. Tanner positioned watching a movie as the longest duration of the four experiences. I asked Tanner why he thought watching a movie took so long. His response is shown in Excerpt 4.5.

Excerpt 4.5 (August 2020): Tanner's Description of the Duration of his Accumulating Activities while Watching a Movie

1. T (Tanner): Well, movies take, like, 15 hours, **and, more**, that's more than 15 minutes (his previously specified length of eating lunch).
2. A: Why do they take so long?
3. T: Because they do.
4. A: What happens during a movie that makes it take so long?
5. T: It does. You can eat popcorn and drink stuff.
6. A: Oh, so because you have the movie **and** the popcorn **and** the drink it takes longer?
7. T: [nods affirmingly]

As Tanner described the duration of watching a movie, he seemed to reflect on more activities than just his watching. Eating popcorn and drinking are two activities that can accompany watching a movie, and I infer that Tanner conceived of them as disparate parts of his experience. When re-presenting the duration of a movie, I conjecture that Tanner did not re-present watching the movie, eating popcorn, and drinking being done simultaneously (Piaget, 1969). Hence, Tanner's long duration seemed to result for the accumulation of distinct activities. Additionally, I found no evidence that Tanner was quantifying the sub-durations of each activity within the overall experience. Thus, I infer that Tanner has yet to unitize duration as an accumulation of activities completed (Steffe, 1991; von Glasersfeld & Steffe, 1985). Rather, I conjecture that for him the more activities within a single experience necessarily produces a longer overall duration. I find this similar to my participants' conception of duration as a consideration of magnitude, which I will discuss in the next section.

I also asked 4-year-old Cody about the duration of his experiences from the Tillman and Barner (2015) study. Much like Tanner, Cody seemed to conceive of duration as an accumulation of activities as he explained how long it took him to brush his teeth. Excerpt 4.6 presents his response.

Excerpt 4.6 (August 2020): Cody's Description of the Duration of his Accumulating Activities of Brushing his Teeth

1. C (Cody): A very, a **very long**, uh, a little time.
2. A: A very long or a little time?
3. C: Uh, a **little** time!
4. A: Why is it a little time? How do you know?
5. C: Because sometimes my mom just goes brushing all of my, just my teeth?
6. A: Can you say that one more time?
7. C: Um, sometimes she, uh, just brushes my teeth.
8. A: How do you know that takes a little time?
9. C: Because sometimes she brushes my tongue **and my** teeth.
10. A: Oh, so when she, when she brushes your tongue, does that make it take longer or shorter?
11. C: It's takes, we need to brush it **a lot**, we need to brush our tongue **and** our teeth because when food touches our tongue then my mom needs to brush it.
12. A: And when she brushes your tongue, does that make it take longer to brush your teeth or shorter?
13. C: Shorter, uh [hesitates for 3 seconds as he scratches his head]
14. A: How come?
15. C: [hesitates as he looks around] Because, um, **it just**, uh takes, um [hesitates for 5 seconds] wait a second [walks away from the camera]

To me, Cody seemed to be considering the accumulating activities of brushing his teeth as he reflected on the duration of his experience. From his initial hesitations—repeating himself (Line 1), changing his measurement (Line 1) saying "uh" (Lines 1, 3) and the upward inflection of his response (Line 5)—I infer that Cody seemed to be re-presenting his experience as he was answering my questions. About halfway through his response (Line 9), Cody seemed to crystallize his re-presentation as he clearly described that brushing his teeth takes a little time because his mom brushes both his tongue **and** (his emphasis) his teeth. For Cody, the experience of brushing teeth seemed to involve two possible, separate activities—brushing his tongue and brushing his teeth. Thus, I infer that he conceived of the duration of this experience as an accumulation of both activities.

Cody's reasoning that it took less time when his mom brushed his tongue as well as his teeth (Line 11) seemed paradoxical to me (Tzur, 2019). Generally, performing several activities during a single experience would necessarily make the overall duration of the experience longer, as they would be done sequentially, not simultaneously (Piaget, 1969). Cody's hesitation about his accumulating activities (Line 11, 13) may reflect his consideration of the sub-durations of brushing his teeth and tongue within the overall duration of his brushing teeth experience (Piaget, 1969). From this, I surmise that Cody was reasoning about duration in an intuitive manner (Piaget, 1969). He seemed able to re-present his experiences, and was developing early intensive quantification (Russell, 2008) of longer and shorter. However, similar to Shelby's description of vacuuming (Excerpt 4.1), Cody's non-standard, non-enumerated conception seemed to indicate that he was yet to unitize the disparate activities (Steffe, 1991; von Glasersfeld & Steffe, 1985), hence duration, through his reflection on his accumulating activities.

Cody and Kyla (both age 4), and Mick and Tanner (both age 5) all seemed to conceive of the duration of at least one experience through a reflection on accumulating activities alone. In other words, when describing their experiences, I infer that there were specific durations that were explained by the 4- and 5-year-old participants through the accumulation of activities with no apparent consideration of other attributes of their experiences. This, unlike Shelby (from Excerpt 4.1) who drew from her accumulated activities, her consideration of gross quantities, *and* her own exertion as she conceived of the duration of vacuuming, was a notable feature of the younger participants—with one exception. During Shelby's initial interview, I asked if she could think of something that took a short time. She looked around for quite a while, and then described brushing her hair, shown in Excerpt 4.7.

Excerpt 4.7 (August 2020): Shelby's Clear Description of the Duration of her Accumulating Activities of Brushing her Hair

1. S (Shelby): [hesitates for 4 seconds] Hmm [hesitates for 3 seconds as she looks around] mmm, maybe when you, um, need to brush **your hair**.
2. A: How do you know that that takes a short time?
3. S: Because my mom, she brushes it and then she puts it up in a ponytail **and then it's done**.
4. A: Okay, so she brushes it and then puts in a ponytail and it's done and so that's short time?
5. S: [nods affirmingly]

Shelby's clear retelling of brushing her hair showed, what I see as, the most well-defined example of duration as reflection of an accumulation of activities completed from my participants. This is because of how clearly Shelby sequenced the activities of her experience (Piaget, 1969). There were distinct activities (sub-durations) involved in the overall short

duration of Shelby's experience: her mom brushes her hair, *then* puts it in a ponytail, *then* it's done (Line 3). Her experience was clearly bound (von Glasersfeld, 1981) in her re-presentation, by her pronouncement "it's done" (Line 3), and I infer that Shelby's emphasis on these two words highlighted her perceived short duration of the experience. Shelby's described activities were sequential, though non-standard, and non-enumerated. Thus, while I view this as a model description of accumulating activities, Shelby did not seem to demonstrate a unitized conception of duration (Steffe, 1991; von Glasersfeld & Steffe, 1985) during this re-presentation.

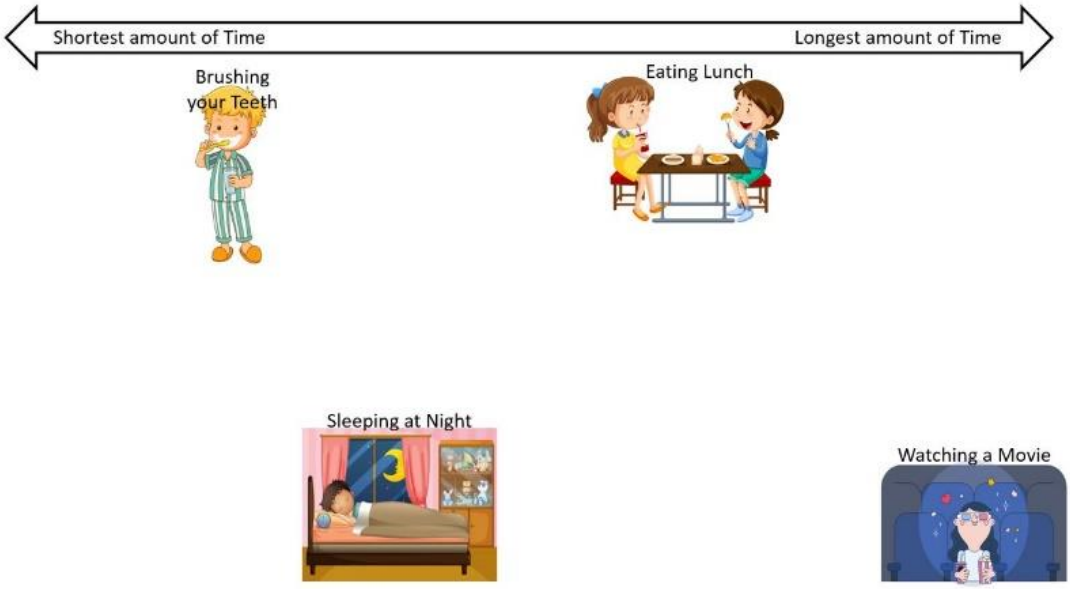
This excerpt was the only example from my 6-year-old participants of duration as an accumulation of activities completed that did not seem to incorporate other measurable attributes, such as magnitude or effort. I conducted this interview in August 2020, prior to Shelby's start of first grade. During her follow-up interview, a month later and after she had begun first grade, Shelby seemed to conceive of accumulating activities in conjunction with other measurable attributes of her experiences, such as her description of vacuuming (Excerpt 4.1).

Another first grader, Lennon, seemed to consider multiple attributes of his experience as he organized the four experiences noted by Tillman and Barner (2015) during his initial interview. His description, however, was unlike any other participant. As Lennon positioned the experiences, he was the only participant to explicitly attend to the relative distances/relationships between the durations of the experiences. For example, Lennon began by considering the duration of brushing his teeth. He explained that it was short, but not the shortest possible duration that an experience could be. So, he positioned it *near* the far-left end of the open-ended durational "timeline" but specified that it should not be all the way down. Next, he reflected on eating lunch, stating "the lunch would take not too much long," which I conjecture to mean he

was comparing the duration of eating lunch against his established "short" duration of brushing his teeth. When Lennon situated lunch on the timeline, however, he gave a sizable gap between the two experiences, as shown in Figure 6.

Figure 6

Lennon's In-Progress Durational Number Line



Note. Lennon first positioned brushing his teeth (Graphics RF, n.d.b), then eating lunch (Graphics RF, n.d.c). He had not yet considered the other two experiences (sleeping at night (Graphics RF, n.d.a) and watching a movie (Twilightmoon, n.d.))

I asked Lennon why he placed eating lunch where he did. His response is presented in Excerpt 4.8.

Excerpt 4.8 (August 2020): Lennon's Description of the Short Duration of Eating Lunch

1. L (Lennon): Mmm, probably because it, kind of **feels** like it's short.
2. A: How come?
3. L: If you, if you shove lots of things in your mouth [pretends to shove food in his mouth] and it'll be faster, but you'll have to swallow **a lot**.

Lennon's initial reasoning, that lunch "feels" short (Line 1), was a description that four of the participants conveyed during their interviews. Specifically, the conception that the participants might perceive duration by how it *feels to be in the experience* as they re-present and reflect on their experiences. This might correlate with conceiving of duration through sensorimotor actions, as I explained in Chapter II (von Glasersfeld, 1995), where duration is intuitively perceived to reside in the experience itself. However, because I could not articulate feelings in my analysis, I chose to omit these excerpts from my analysis.

As Lennon reflected on his assertion that the duration of eating lunch felt short, I conjecture that he conceived of the short duration through several attributes, specifically his unique lunchtime activities (coded green), the quantity of food being eaten (coded blue), and the rapidity of his actions (coded red). It seemed that to Lennon, his activities of quickly shoving food in his mouth and swallowing (Line 3) bound his experience, causing it to be short, despite his consideration of the large quantity of food. To me, this demonstrated a stronger conception of Lennon's accumulation of activities impacting the duration of his experiences, than his consideration of the gross quantities of food or the rapidity of his experience. That is, Lennon seemed to foreground his accumulating activities while being aware of the other two attributes of his experience as he conceived of the duration.

Six-year-old Easton also seemed to conceive of multiple attributes as she described her experience of building various things with LEGO pieces. However, unlike Lennon, Easton seemed to correlate the size (gross quantity) of her build along with her accumulated activities as she reflected on the overall duration of her experience.

During her follow-up interview, Easton told me about a new LEGO set she had just built. I asked if she always followed the set directions or if she sometimes made up her own designs. She enthusiastically explained that she had designed her own couches out of LEGO for her Barbie dolls, "I made a cute little and big couch." Excerpt 4.9 shows our continued conversation.

Excerpt 4.9 (September 2020): Easton's Description of her Accumulating Activities while Building LEGO Couches

1. A: Can you tell me about your couches that you built?
2. E (Easton): Um, one is, like, **long** and one is short and they're **completely** different.
3. A: What do you mean they're completely different, can you describe them?
4. E: So, um, one I put, like, flat tiles on and one, the other, **big one,** is, like, just bumpy.
5. A: Very cool. Which one do you like better?
6. E: I think the big, long one, it's, like, fancy and it has, like, pink, purple, red, brown. I mean— [hesitates for 6 seconds]
7. A: Do you think you put more time into making that one?
8. E: Yeah.
9. A: How come?
10. E: Because it was **longer** and I just sort of, like, took a few pieces and then I wanted to build a couch because **first** I wanted to build a mirror that you looked into, like a, a lipstick stand, where you get ready?
11. A: Yeah.
12. E: But then I'm like, ooh, a couch would look good.
13. A: So, your idea changed?
14. E: Yeah, um, and then I just started building what looked like a couch and there was a lot one layer [sic], but I wanted it to be longer so they can, like, lean back or something, so I built, like, another layer.

15. A: Awesome!

16. E: And, like, I had to make it wider. And then— [hesitates for 5 seconds]

17. A: How did you know you were done building?

18. E: Because it looked pretty much like a couch.

In Line 10, Easton reasoned that the duration of making the bigger couch took longer than the smaller couch because the bigger couch was longer. Here, I infer that she attended to the relative size of the couch to conceive of the duration of her experience. However, as she described her re-presented experience, she seemed to elaborate more on her building process—as she coordinated her accumulating activities in conjunction with her consideration of gross quantity. Figure 7 provides a narrative visualization (Segel & Heer, 2010) of Easton's progressive description of building the LEGO couches, specifically her attention to the activities completed while building the big couch *and* her consideration of the amount of LEGO pieces and the increasing size of the couch throughout her experience.

As Easton re-presented her experience of building the bigger couch, she seemed to conceive of her accumulating activities (taking pieces (Line 10), changing her mind about what to build (Line 10-12), starting to build the couch (Line 12), adding a layer (Line 14), and making it wider (Line 16)) in coordination with her consideration of a gross quantity (she had to add layers (Line 14) to made it longer (Line 14) and wider (Line 16)), in such a way that her activities resulted from her consideration of gross quantities.

Figure 7

Narrative Visualization (Segel & Heer, 2010) of Easton's Description of Building LEGO

Unlike Lennon in Excerpt 4.8, Easton's consideration of gross quantities seemed to explain her accumulating activities, which she then used to quantify the duration of her experience. I could not say if this might be a marker (Tzur, 2019) of a more developed durational quantification, however, it seemed like notably different reasoning.

Summary of Duration as an Accumulation of Activities Completed

The data presented here were examples of re-presentations that I inferred demonstrated my participants' interiorization of duration as an accumulation of activities completed. Through my analysis of this theme, I found two distinct conceptions that my participants seemed to use when conceiving of the duration of an experience as an accumulation of activities: (a) repeating a single activity multiple times, and (b) completing several, different activities within a bounded experience (von Glasersfeld, 1981). In both cases, I infer that my participants conceived of the duration of their experience as being partitioned into smaller, sub-activities (sub-durations). To attribute such a logical conception, it is imperative that the participants conceived of sub-activities as having a shorter duration than the overall duration of their experience (Piaget, 1969).

When my participants conceived of duration as an accumulation of activities completed, I conjecture that they were not necessarily quantifying their experience using any discernible, standardized units (Steffe, 1991; von Glasersfeld & Steffe, 1985). In other words, from the data, I saw no evidence of my participants' conservation of speed, transitivity, or unit iteration (Long & Kamii, 2001). Instead, the participants seemed to be quantifying the duration of their experience by bounding their experiences (von Glasersfeld, 1981) through their accumulating activities to conceive of the duration of their experience as a perceptual unitary item. This, perhaps, is a precursor to Long and Kamii's (2001) description of durational measurement (as I described in Chapter I and II), and an extension of von Glasersfeld's (1981) attentional model. I

next turn to examples of the participants' conception of duration as a consideration of a gross quantity.

Duration as a Consideration of a Gross Quantity

To illustrate duration as a consideration of a gross quantity, I used Wolcott's (1994) *description*, *analysis*, and *interpretation* to provide evidence of the participants' conceptions of duration reasoned through reflections on qualitative (non-enumerated) and quantitative magnitude, relative size, and distance. As demonstrated through Excerpts 4.8 and 4.9 in the previous Accumulating Activities section, I conjecture that my participants often reflected on multiple attributes of their experiences simultaneously. For example, as Easton (Excerpt 4.9) described her experience of building LEGO couches, she seemed to coordinate her conception of the relative size of her build with her accumulating activities of adding more layers.

Five-year-old Mick provided a different example of this coordination of conceptions, as she reflected on her experience of baking muffins. What I found most striking about Mick's conceptions was that I infer she reflected on magnitude during her initial interview to conceive of a long duration but seemed to shift her reflection to her accumulating activities as she described baking muffins in comparison to baking cookies during her follow-up interview. Excerpt 4.10 presents Mick's description during each interview.

Excerpt 4.10 (August and September 2020): Mick's Shifting Reflection of the Duration of Baking Muffins

Initial Interview:

1. A: Mick, can you tell me, what's something that takes a long time?
2. M: Um, it takes a **really long time** for muffins to be done.
3. A: How do you know it takes a really long time for muffins to be done?

4. M: Because there's **steam**, there's a lot of **steam** in it.

Follow-up Interview:

5. A: Do you remember when we talked last, you said that you made muffins?

6. M: I made it in a **long** time.

7. A: Yeah, that's what you had said. Did the muffins take you longer than the cookies, or shorter?

8. M: Umm [hesitates for 5 seconds] shorter.

9. A: So, the muffins were (a) shorter (time)?

10. M: Yeah.

11. A: How come?

12. M: Because, because we put them in the oven and then we take them **out**, they were very fast.

13. A: So, you put it you put the muffins in the oven and then when you take them out, it's really fast. Last time you told me that it took a long time for your muffins to cook because you had to wait for all the steam to come out of them.

14. M: Oh, you just have to poke **a hole** in them, and the steam gets out.

15. A: Oh, so you can make it go faster?

16. M: Yeah [nods affirmingly].

17. A: If you didn't poke a hole, would it take longer?

18. M: [nods once as she says] Yes.

Mick seemed to reflect on the magnitude of steam ("a lot," Line 4) in the muffins during her initial interview. During this description, she did not explicitly bound her experience (von Glasersfeld, 1981), but rather seemed to focus solely on the amount of steam to explain the long duration of her experience. When I asked Mick to compare the duration of baking muffins to

baking cookies during her follow-up interview, I infer that she reflected on the activities she completed while baking. To begin, Mick clearly described the boundaries of her experience (von Glasersfeld, 1981) through her activities, “we put them in the oven and then we take them out” (Line 12). Between these two boundaries, Mick seemed to reflect on the rapidity of the baking, “they were very fast” (Line 12), which I would not characterize in the same way as my future code, duration as the result of rapidity (exertion) because the rapidity is not of Mick’s actions, but of the muffins.

I then asked Mick about the steam that she had mentioned during her initial interview, as it was the quantity of steam on which she seemed to reflect to quantify the duration of her experience. Her response, “you just have to poke a hole in them” (Line 14), seemed, again, to focus on her activity rather than the amount of steam. I find this to be notably different than how Easton (Excerpt 4.9) conceived of a gross quantity and accumulating activities simultaneously. To Easton, it seemed that the gross quantity produced her accumulating activities, and vice versa. Mick, on the other hand, did not seem to acknowledge the correlation between her two conceptions. I conjecture that Mick conceived of the duration of her experiences as an accumulation of activities completed or as a consideration of a gross quantity, but not both simultaneously. Neither Easton nor Mick seemed to indicate that they conceive of the duration of their acuminating activities as a unitized measure (Steffe, 1991; von Glasersfeld & Steffe, 1985). Rather, they seemed to describe duration as a consideration of a gross quantity. All seven of my participants reflected on a gross quantity in a variety of ways, as I will now present.

Consideration of Magnitude

When reflecting on magnitude, all participants in my study considered the amount, or quantity, of a particular aspect of their experience. Some of the verbalizations had general,

qualitative magnitudes (e.g., Mick's "a lot" from Excerpt 4.10, Easton's "a few" from Excerpt 4.9). Piaget (1965) characterized these as global comparisons, an early stage in the development of number. Other descriptions from my participants named specific, quantitative magnitudes (e.g., Shelby's "one room" from Excerpt 4.1). From the seven participants, I found that quantitative magnitudes were only described by the 6-year-olds (Shelby and Easton), while all seven children seemed to reflect on qualitative magnitudes.

Qualitative (non-enumerated) Magnitude

Similar to Mick, 4-year-old Kyla described both her accumulating activities *and* gross quantity (though not necessarily in coordination, as Easton had in Excerpt 4.9) as she reflected on the duration of painting pictures for her family. She excitedly told me about the two paintings she had done, one for her mom and the other for her entire family. Excerpt 4.11 presents her description of Kyla's painting for her mom.

Excerpt 4.11 (September 2020): Kyla's Consideration of the Magnitude of Dots and Lines she Painted

1. A: Oh, which picture took you longer to paint, the one for your mom or the one for the whole family?
2. K (Kyla): The one for my mom.
3. A: Why did it take longer?
4. K: Because, I had to do **so many** dots.
5. A: Can you tell me (more) about painting the picture for your mom?
6. K: So, like, there were **so many lines** and, like, I drawed [sic] them straight and then all the dots—
7. A: So, you painted the lines first and then the dots, how did you know that you were done with your painting?

8. K: [hesitates for 4 seconds] Because.

9. A: How did you know? Did your mom say you were done, or did you just decide that you were done? Did you run out of paint?

10. K: Well, it's just that there was just no room (on the paper) **more** [sic] to paint.

11. A: Oh, so you ran out of room on the paper to paint?

12. K: Mmhmm.

Kyla's emphasis of "so many" dots (Line 4) and "so many" lines (Line 6), to me, indicated her consideration of duration through qualitative magnitude during her re-presentation. I infer that Kyla conceived of the longer duration of painting for her mom resulting from the amount of lines and dots that she painted—though she was not explicit in naming the exact number of dots or lines made.

Some of Kyla's conception seemed connected to her accumulating activities. For example, in Line 5 she said, "I had to do," showing her recognition that it was her activity that created the dots. She also explained that she drew all the lines straight (Line 6), a clear indication, to me, of her actions during the experience. When Kyla described "so many lines" (Line 6) and "all the dots" (Line 6), it seemed that she considered the magnitude of the lines and dots, not necessarily her accumulating activities of creating them. She clearly described the end of her experience when she ran out of room on her paper (Line 10). To me, this suggested that her consideration of the relative size of the paper was also significant in bounding her experience (von Glasersfeld, 1981). Specifically, Kyla seemed to reflect on a causal relationship (Piaget, 1969) between her repeated actions, the relative space on the paper, and the duration of her experience. So, if her paper had been larger, she might have been able to create more lines and dots, possibly increasing the overall duration of her painting experience. Next, I turn to a

conversation I had with Tanner in which he also seemed to reflect on a qualitative magnitude during his re-presentation.

During his follow-up interview, kindergartener Tanner spoke at length about his experiences from the first month of kindergarten. He described playing *The Floor is Lava* and *Tiger Chase* with his friends at recess, both of which he seemed to enjoy. Tanner also explained different building activities he did during centers time. One specific activity Tanner talked about was Marble Run (a sample Marble Run is shown in Figure 8).

I asked Tanner to tell me about Marble Run, which he explained by stating, "I used all the pieces, and I made a very big, big Marble Run." I was interested in his reflection on the duration of a marble to complete his course because he specified that he used all the pieces. Tanner's response is shown in Excerpt 4.12.

Figure 8

Marble Run Example

Excerpt 4.12 (September 2020): Tanner's Description of the Qualitative Magnitude of Marbles used in his Marble Run

1. A: When the marble went down, did it take a long time for it to get through the whole course?
2. T (Tanner): No.
3. A: No, how come?
4. T: [hesitates briefly, then nods affirmingly]
5. A: Oh, so it did take a long time?
6. T: Yes.
7. A: Why did it take a long time?
8. T: Because of how many **marbles** we setted [sic] in, we, like, set in **all the** marbles.
9. A: Oh, my goodness, so you didn't just do one you did all of them at once?
10. T: Uh-huh.

I infer that Tanner's conception of the duration of his experience was reflected through his consideration of the magnitude of marbles he sent through the course (Line 8). Tanner quantified the duration of the marbles through the course as *long*, "Because of how many marbles we setted [sic] in, we like set in all the marbles" (Line 8). Here, Tanner seemed to be equating a large quantity ("all the marbles") with a large duration (long). I conjecture that Tanner's reasoning was analogous to Piaget's (1969) identification of children at stage I of durational reasoning (as I described in Chapter II) with Tanner's reasoning being a quantification of magnitude, not necessarily duration. In other words, Tanner seemed to be conflating the space of his experience with the time (Piaget, 1969).

Through my initial question (Line 1), I intended to elicit Tanner's reflection of one ("the") marble through the course. The implied bounding of his experience (von Glasersfeld, 1981) being between setting the marble in the top of the run and when the marble reached the bottom. I infer that Tanner assimilated my question in such a way that the magnitude of the marbles impacted his perceived duration of the experience, and logically so. More marbles *would* necessarily create a longer duration because the marbles can only proceed successively through the course (i.e., one at a time).

I found Tanner's reasoning striking because of his previous re-presentation of watching a movie (Excerpt 4.5). When Tanner described a movie as taking a long time, he specified multiple activities (eating, drinking, and watching) that he would, most likely, complete simultaneously—thus, not necessarily creating a longer duration. Tanner's conception of the magnitude of marbles creating a longer duration seems notable to me, because the individual marbles *must* proceed through the run successively (not simultaneously), thus the longer duration is implicit—but the individual activities of the movie would most likely be simultaneous, thus the longer duration is not necessarily so. It may be, therefore, that there is a conceptual difference between the magnitude of activities (sub-durations) and the magnitude of tangible objects of an experience.

Tanner, Kyla, and Mick all seemed to consider qualitative magnitudes to conceive of the duration of their long experiences. "A lot" (Mick), "all" (Tanner), and "so many" (Kyla) quantified both the magnitude *and* the duration of those participants' experiences and may be reflective of these 4- and 5-year-olds' intuitive conceptions of time (Piaget, 1969). While all seven participants quantified duration through their consideration of qualitative magnitudes during at least one experience, two of the 6-year-old participants (Shelby and Easton) also

numerically quantified specific attributes of their experience as they reflected on the duration of their experiences.

Quantitative Magnitude

In Excerpt 4.1, I described a conversation I had with 6-year-old Shelby about the duration of vacuuming different parts of her house. During her description, Shelby described vacuuming just *one* room. She stated that one room would take about a minute to vacuum, which would be a short amount of time. It seemed reasonable to me that Shelby might conceive of a minute as a short duration, possibly due to how this term is used colloquially. However, what interested me was Shelby's consideration of vacuuming *just one* room—it "probably takes, like, a minute because it's, like, one room." Here, she numerically quantified the number of rooms, rather than speaking generally about a *little* amount of or a *few* rooms. I surmise that she could conceive of partitioning the overall duration of vacuuming into discrete durations by specific room, though these may not be standardized, as described by Steffe (2013). It should also be noted that it may not actually take only 1 minute to vacuum one room—especially when considering Shelby's retelling of the effort of vacuuming, the weight of the vacuum, and the different sizes of the rooms; however, to me, there seemed to be more reflection on the duration of this minute than, say, how Reece described the 5 minutes of brushing her teeth in Excerpt 2.2 (in Chapter II).

Like Shelby, Easton also described duration using standard units. During her initial and follow-up interview, 6-year-old Easton named specific, standard-unit durations. She then went on to describe the quantitative magnitude of an attribute of those experiences as she described duration. During my first conversation with Easton, I asked her to tell me about her experiences that took a short amount of time. She explained that brushing her teeth took a short amount of time because it only took 2 minutes. I asked if she could think of something else that took about

the same amount of time. Easton hesitantly stated that reading one chapter of a book might also take a short amount of time. Based on this comparison, I asked Easton to reflect on this duration, shown in Excerpt 4.13.

Excerpt 4.13 (August 2020): Easton's Quantification of the Duration of Reading

1. A: Okay. Earlier you told me that brushing your teeth was a short amount of time and it takes 2 minutes. Does that mean that reading one chapter of your book would take you 2 minutes?

2. E (Easton): Uhh, maybe, maybe if it's, like, two **pages** or something.

…

3. A: That's cool! How long would it take you to read a book that had 50 pages?

4. E: Uh, maybe **an hour** [shrugs shoulders]?

5. A: How come?

6. E: Uh, because, um, 50 pages are **a lot** of pages and an hour's a lot amount of time [nods].

From Easton's first response, "Uhh, maybe, maybe if it's, like, two pages or something" (Line 2), it seemed, to me, that she was simultaneously coordinating the magnitude of one chapter and re-presenting her experience of reading, to quantitatively describe the duration of her experience. Unlike in Reece's case (Excerpt 2.2, Chapter II), I found this to be a significant mark of Easton's reasoning, where the standard units seemed to have some durational meaning to her.

Easton's durational quantification seemed to change slightly when she described reading 50 pages of a book as taking about an hour. Here, it seemed that Easton might not have conceived of reading each page as taking an equal duration (as in reading one page per minute). Instead, she described the qualitative magnitude of "a lot of pages" (Line 6) taking an hour because an hour is "a lot of time" (Line 6). I wonder if Easton's conception of 50 pages taking 60 minutes (versus her earlier reasoning that two pages might take two minutes) might suggest her

concept of number, as I described in Chapter II (Steffe & von Glasersfeld, 1985; Ulrich, 2015), her conception of smaller units of time versus larger units of time, as I explained in Chapter III (Reisman, 1971), or some other conception, such as the effort (exertion) of reading for an hour versus a few minutes.

Explaining the duration of experiences through standard durational units (such as an hour) was a concept that several other participants struggled to explain. Excerpt 4.14 presents two such examples.

Excerpt 4.14 (August 2020): Lennon (from Chapter III, Figure 2) and Tanner's Reflections on Standard Durational Units

Lennon's Reflection:

1. A: What's an hour?
2. L (Lennon, age 6): Uh, it's a thing that there's **multiple** minutes.
3. A: How many minutes are in the hour?
4. L: Uh, probably, like, **50 minutes? 60?!**
5. A: Which one is it, 60 or 50?
6. L: Either 50 or 60 [shrugs shoulders].

Tanner's Reflection:

1. A: How is an hour different than a minute?
2. T (Tanner, age 5): **Because** it is.

Both Tanner and Lennon named standard durational units as they described their experiences. However, neither presented evidence of reasoning about how these standard units might quantify the duration of their experiences. Thus, when considering a quantitative magnitude, I did not consider the participants' use of standard units to reflect a "quantified

magnitude." With the prevalence of standard durational units in the United States, as well as the possible impact of colloquial uses of these standard units (e.g., "hang on a minute," as I described in Chapter II), it seemed challenging to infer how a young child might conceive of those durations.

What I found significant as I looked across the data was that the only three occurrences of duration as a consideration of a quantitative magnitude happened to also have a specific standard durational unit mentioned during the description. For example, this was shown when Shelby stated that it would probably take a minute to vacuum one room (Excerpt 4.1), or when Easton explained that two pages would take about two minutes to read (Excerpt 4.13). These were not the only instances of the participants naming standard units during their interviews; however, these were the only instances where the participants seemed to re-present and describe the duration of their experiences while using those terms.

Easton described the duration of another experience using standard durational units while considering a quantitative magnitude. During the previously mentioned LEGO discussion (Excerpt 4.9), Easton told me about the Moana LEGO set she had just built. She was very excited to talk about the unique colors of the pieces and how the set included a LEGO version of Moana in a floral sarong and even Moana's little pig, Pua, shown in Figure 9. I asked Easton about building her Moana LEGO. Easton's description is presented in Excerpt 4.15.

Excerpt 4.15 (September 2020): Easton's Consideration of a Quantitative Magnitude to Conceive of Standard Durational Units

1. A: Have you built the whole Moana set already?
2. E (Easton): Yeah.
3. A: How long did it take you to build?

4. E: Maybe [hesitates briefly] 5, 10, **5 to 10** minutes.

5. A: Was that a long time or a little time?

6. E: Mm, a little.

7. A: How do you know it was a little?

8. E: Because, well, it's, like, [hesitates for 5 seconds] because I thought there was **another page,** but it was, like, none [pretends to flip a page, then shrugs]

9. A: So, you were expecting to do more but then you ran out of pages?

10. E: Yeah.

Figure 9

Sample of Easton's Moana LEGO Set

I conjecture that Easton's re-presentation of flipping the pages in the LEGO guide and expecting to find "*another*" page but there were "*none*" left (Line 8), demonstrated a consideration of a quantitative magnitude of duration. Her conception of "none" (Line 8) seemed to indicate a lack of magnitude, or zero pages left, which seemed to explain her quantification

that 5 to 10 minutes was a little time (Line 6). I also infer that there was also an inherent accumulation of activities described in Easton's account of flipping pages.

To me, Easton seemed to clearly bound her experience (von Glasersfeld, 1981) through her reflection on zero pages, not necessarily her activity of building the set. Easton did not indicate that she considered the duration of each page as standardized in such a way that she could iterate the duration spent on each page to create the overall duration of following the guide. It seemed to me that, for Easton, duration as a consideration of quantitative magnitude was yet to be unitized (Steffe, 1991; von Glasersfeld & Steffe, 1985).

Building LEGO sets were a common experience shared by several of the children. But what attribute of the LEGO building experience the child reflected on (whether magnitude of pieces or relative size of the build) differed across the participants' re-presented accounts. This is shown next.

Consideration of Relative Size

As previously described in Excerpt 4.9, while 6-year-old Easton recalled her experience of using LEGO to build couches for her Barbie dolls, she seemed to reflect on her accumulating activities in coordination with the relative size of her couches. Specifically, Easton described adding additional LEGO to increase the overall size and width of her couch. Because the size of her couch increased, Easton reasoned that the duration of her experience was longer, indicating her recognition of the causal relationship (Piaget, 1969) between her actions of making the couch bigger and the overall duration of her experience. It was interesting that Easton's re-presentation focused on the size of the couch, but not necessarily on the amount of LEGO pieces she used to make it.

Preschooler Cody also described several experiences of building LEGO. During his initial interview, he and I had an extended conversation about his different LEGO builds. Throughout this conversation, shown in Excerpt 4.16, Cody seemed to simultaneously consider both magnitude and relative size as he described several different LEGO builds.

Excerpt 4.16 (August 2020): Cody's Consideration of the Relative Size of his LEGO Creations

1. A: Can you tell me something that takes a long time?
2. C (Cody): Uhh [hesitates for 6 seconds and looks around] um, uh, um, building!
3. A: Building what?
4. C: LEGOS [sic] that take a **long** time!
5. A: How do you know it takes a long time?
6. C: Because that LEGO up there [points to shelf] that takes a long time to build.
7. A: But why? Why does it take a long time?
8. C: Uh because it has **a lot** [spreads arms wide] of pieces.

In Line 6, Cody pointed to a new LEGO set he was planning on building with his dad. He anticipated that this set would take a long time to build because there were a lot of pieces (Line 8). I infer that Cody based his anticipation on his interiorization of past LEGO building experiences, as I described in Chapter II (Norton et al., 2018; Olive, 2001). Cody classified the magnitude of LEGO pieces in the set ("a lot") and reasoned that a large magnitude would cause a long duration, though he did not seem to bound his experience (von Glasersfeld, 1981).

I was curious about Cody's consideration of the magnitude, so I asked him to anticipate what might happen to the duration of his experience if the LEGO set had less pieces (Excerpt 4.16 continues below).

9. A: Oh, so if it had only a few pieces would it take as long?

10. C: No. But if it was a tiny little one, then it would be just the **rocket ship** in the box.

11. A: Would it take a long time to build just the rocket ship?

12. C: Noooo! Wait, wait; we have a real rocket ship **downstairs**!

13. A: Cool!

14. C: It's made out of LEGOS [sic]! And it's **big**!

15. A: Did your dad build it (he had previously spoken about his dad's building)?

16. C: Yeah.

17. A: Did it take him a long time to build?

18. C: No [hesitates briefly] um, yeah.

19. A: Oh, it did take him a long time?

20. C: Yeah.

Cody's consideration of a gross quantity seemed to switch from the qualitative magnitude of the LEGO pieces to the overall size of the build. I found this to be similar to Easton's account of building the LEGO couch. When I asked about Cody's use of a few pieces (magnitude, Line 9) he answered about the "tiny, little" size of a rocket ship (relative size, Line 10). He seemed to quantify the duration of this potential experience of building "just the little rocket ship" as taking a short time (Line 12) based on his consideration, again, of the relative size of the ship.

Cody went on to compare his "tiny, little" rocket ship to a big rocket ship that his dad had built (Line 15-2). Here again, Cody seemed to conceive of the duration of his building experiences as corresponding with the relative size of the build—big rocket ship, longer duration (Line 14-20).

I then asked Cody to compare his long duration of LEGO building with another activity. Cody continued his reflection of the size of the big LEGO rocket ship:

21. A: Can you tell me something that takes longer than building LEGO?

22. C: Uh, um, a **big** LEGO ship!

23. A: So, a bigger LEGO would take longer?

24. C: Yeah! [nods enthusiastically]

25. A: How come?

26. C: Like, that **big** LEGO ship that we have downstairs, that took longer. But it was just little, it was this size [puts arms up about shoulder width apart] first, can you see how size it is?

27. A: I can!

28. C: And then, then we **add one more piece on** and then the astronauts, and then it was this tall [spreads arms as tall as he could] after we put **one more on**.

29. A: Oh, my goodness (in awe of his utterance)! So, if a LEGO is bigger, it will take you longer to build?

30. C: Yeah.

Cody's reflection that the ship was little—with his gesturing (Hornbein, 2015) of the size (Line 26)—then adding one more piece which made it taller (Line 28), seemed similar to Easton's reflection on building the LEGO couch in Excerpt 4.9. Adding more LEGO pieces (magnitude) necessarily made the size of the LEGO ship bigger (relative size) which took longer to build (duration). To me, this also seemed like Kyla's description of painting the picture for her mom in Excerpt 4.11, where a bigger piece of paper would have allowed her to paint more, which theoretically would have meant painting for a longer duration. Unlike Kyla and Easton's re-presentation of this causal relationship (Piaget, 1969), Cody seemed to reflect more on the size

of the LEGO than his activities of creating it, as he explained the duration of his experiences. Table 3 presents a comparison of the three participants' explicit causal reflections.

Table 3

A Comparison Between Cody, Easton, and Kyla's Cause and Effect (Piaget, 1969) Descriptions

Participant	Excerpt	Cause	Effect	Inferred Duration
Cody	4.16	"then we **add one more piece on** and then the astronauts"	"then it was this tall [spreads arms as tall as he could]"	A bigger LEGO causes a longer duration
Easton	4.9	"I wanted it to be longer"	"so I built, like, another layer"	The bigger couch took longer to build "Because it was **longer**" and "**first** I wanted to build a mirror… But then I'm, like, ooh, a couch would look good… then I just started building what looked like a couch"
Kyla	4.11	"there were **so many lines** and, like, I drawed [sic] them straight and then all the **dots**"	"there was just no room **more** to paint"	"The (painting) for my mom" took longer to paint "Because, I had to do **so many** dots"

I turn to an excerpt from first grader Lennon, as he implicitly described a causal relationship between the size of a circular path he traveled and his reflection of the duration of traveling it. In Excerpt 4.17, Lennon elaborated on this relationship as he considered running in a small circle taking a short amount of time.

Excerpt 4.17 (August 2020): Lennon's Consideration of the Duration of Running in a Circular Path

1. A: Can you think of something else you do that takes a short amount of time?
2. L (Lennon): Umm [hesitates briefly] uh, running.
3. A: Running, how far would you—
4. L: Running in a circle that's **a tiny circle**.
5. A: Why not running in a big circle?

6. L: Umm, cause the tiny circle is probably, like, **really fast**, cause, like [draws a small, quick circle in the air and makes a zippy sound effect]

7. A: Okay, and what would a big circle be like?

8. L: Umm, probably be, like [slowly draws a large circle with his hand] **this big**.

9. A: Okay, would it take you longer or shorter to run in **that** big circle?

10. L: [hesitates for three seconds] You probably know the answer, **longer**!

11. A: Why would it take longer?

12. L: [hesitates for 5 seconds] Since it's a bigger, the bigger, **the slower.**

13. A: The bigger, the slower?

14. L: The bigger, **the longer!**

15. A: What is the difference between the bigger the longer and the bigger the slower?

16. L: [hesitates for eight seconds and rubs head] Yeah, this is, hard, I don't know.

After stating that running would take a short time, I began to ask *how far* would you run in a short time? Based on my own experiences, the primary attributes that *I* would attend to while running would be distance (which I will discuss in the next section) and rapidity (which I will discuss later in this chapter). Lennon, however, assimilated my question in such a way that he seemed to attend to the relative size of the lap he ran, not necessarily the distance covered during that lap.

At the beginning of his description, Lennon seemed to consider the relative size of the tiny circle (Line 4-6) in coordination with his rapidity (Line 6), to quantify the short duration of his experience. I found it interesting that when asked about running larger circles, he again seemed to coordinate size with rapidity though his actions of slowly drawing a larger circle in the

air (Line 8). The boundaries of Lennon's experiences (von Glasersfeld, 1981) were implied through his gesturing, specifically the start of the duration being when he began running in the circle, and the end being his return to the starting point. And while this might also demonstrate an implied consideration of distance traveled, Lennon seemed more attentive to the size of the circle not the distance traveled during his run.

As Lennon drew the circles, I was curious if he attended to the size of his circle or the swiftness of his actions—both in his re-presentation and his gesturing of the experience. I asked, "Would it take you longer or shorter to run in **that** big circle (Line 9)?" By emphasizing "that" I was intending to promote Lennon's reflection on his action of making the big circle slower. He quantified the duration of the larger circle as taking longer. When asked why, he explained, "Since it's a bigger, the bigger, the slower" (Line 12), then changed it to "The bigger, the longer" (Line 14). Based on his response, I concluded that Lennon attended to, and reflected more on, the size of the path run rather than the rapidity of his actions.

I now continue my participants' considerations of gross quantities by turning from their reflection on the relative size to considering distance traveled during an experience.

Consideration of Distance Traveled

Unlike considering magnitude and relative size, few of my participants seemed to reflect on the distance traveled during their experiences. In the course of previously mentioned conversation with 6-year-old Lennon (Excerpt 4.17), I had anticipated that he would consider the distance he ran when quantifying the duration of running in circles (e.g., running a far distance would take a long time, running a short distance would take a short time). Instead, he seemed to reflect on the relative size of the circles he ran (e.g., running in a big circle took a long time, running in a tiny circle took a short time). While there is an implicit connection between the size

of the circle and the distance traveled, Lennon did not seem to re-present distance traveled as he reflected on the duration of his running experiences.

Excerpt 4.17 was a conversation that I had with Lennon during his initial interview. During his follow-up interview, I, again, attempted to engage Lennon in a consideration of the distance traveled during his experience.

Lennon described his experiences of traveling to and from his neighborhood school. During his account, Lennon explained that he had to travel down a hill to get to school, which then meant he had to travel uphill to get home. He was very clear that he much preferred going down to school, which I explicate in the following section. Lennon also explained that sometimes he walked to school and other times he rode his bike, as described in Excerpt 4.18.

Excerpt 4.18 (September 2020): Lennon's Consideration of Distance to Explain Duration of Traveling To and From School

1. A: So, you think it would take five minutes to ride your bike and ten minutes to walk to school?
2. L (Lennon): Yeah [nods].
3. A: Yeah?
4. L: Maybe.
5. A: How far do you think you live from the school?
6. L: Probably either 5 or 10 minutes.
7. A: That's how far?
8. L: Yeah! I **go down** to school.

Here again, I posed my question "How far do you think you live from school" (Line 5) in such a way that I anticipated Lennon might respond by considering the distance of his experience. Instead, he assimilated my question through the duration of his experience, 5 or 10 minutes (Line 6). I found this notable because the distance from his house is an unchanging length, unlike the variable sizes of the circles run that he previously described. Yet, his measurement of this distance was a variable amount, depending on how he was traveling, whether walking (10 minutes) or riding his bike (5 minutes). Lennon's response of how far he lived from school being 5 to 10 minutes seemed to be reflective of Lennon's conservation of length (Piaget et al., 1960). This, in conjunction with the rest of his description, indicated how Lennon coordinated his movement through space with his durational reasoning (Piaget, 1969), such that duration seemed to be a product of his travels.

Shelby, also age 6, considered distance traveled multiple times during her initial interview. The first durational question I asked all my participants was if they could think of something that took a long time (see Chapter III for my complete interview protocol). Shelby conceived of a long duration was through the distance traveled to the park, as Excerpt 4.19 presents.

Excerpt 4.19 (August 2020): Shelby's Description of Distance as she Conceived of the Duration of Walking to the Park

1. S (Shelby): [hesitates for 3 seconds] Umm, probably, uh, going on a walk.
2. A: So, how do you know that it takes a long time to go on a walk?
3. S: Because sometimes we go over, next, (to) **the park** and it's, like, and we go somewhere a little bit **far** with my friend.
4. A: Mmhm.

5. S: And, yeah, that's it.

6. A: How do you know that takes a long time? Is because it's far that it takes a long time?

7. S: Yeah, and it's summer, so, it's also **hot**.

8. A: Oh, so the temperature makes it a long time too?

9. S: [nods affirmingly]

I infer that Shelby's initial hesitation indicated her reflection on, and re-presentation of, past, long walking experiences. She, then, seemed to conceive of the length of her experience through the distance she traveled while going on a walk (Line 3). From her description, I inferred the boundaries of Shelby's experience (von Glasersfeld, 1981) were from leaving her house (start of her experience) to arriving at the park (end of her experience). Shelby's account of going on a walk seemed focused on these boundaries, specifically the park being "somewhere a little bit far away" (Line 3). She did not seem to attend to her activities on the walk, but rather, she considered the distance and, interestingly, the temperature during a specific experience. This interview took place in mid-August 2020, and the weather where Shelby lived was above 90 degrees on average during that time of year. I suspect that it was rather warm throughout the time of Shelby's reflection.

Shelby's conception reminded me of an idea described by Darrell Earnest at the Annual Meeting of Psychology of Mathematics Education in 2019 (Earnest & Chandler, 2019). According to Earnest, if a person were to walk the same-length path twice—once carrying a heavy load and once carrying nothing—the experience would *feel* different to the person each time. Specifically, the individual would perceive walking the path while carrying the load as a farther distance than walking without the load, even though the distance traveled would be the

same. To me, this seemed comparable to Shelby's specific attention on the temperature of her experience. Perhaps if the weather were milder, Shelby would have conceived of the duration of her experience differently because these locations were already bound within her experience. (von Glasersfeld, 1981).

I then asked Shelby if she could think of something that took longer than going for a walk. Excerpt 4.20 shows that, once again, she hesitated before responding.

Excerpt 4.20 (August 2020): Shelby's Consideration of Distance to Explain the Duration of Traveling to Hawaii

1. S (Shelby): Mmm [hesitates for 6 seconds] mmm, I'm thinking when we when we go on a plane; it takes **a long time** to get to the island.
2. A: Which island are you thinking about?
3. S: Uhhm, I'm thinking about **Hawaii** because we went to Hawaii, (it was a) long time to get there.
4. A: How did you know it took a long time to get there?
5. S: Because on the plane I, um, was doing my tablet and I opened the window and it was, like, **far, far** [points past the camera].

Here, Shelby once again seemed to attend to the distance traveled when quantifying the duration of her experience. However, as Shelby re-presented her experience, she seemed to coordinate her consideration of distance traveled with her accumulation of activities completed during her flight (Line 5). She knew the duration of the flight was long because she was able to use her tablet *and* look out the window on the airplane *and* it was far. Shelby's inflection and repetition of the word "far" seemed to emphasize the distance traveled, and I conjectured that an aspect of Shelby's consideration of the far distance traveled was because she crossed over the ocean to get to Hawaii. Shelby lived in a land-locked state, so perhaps she correlated seeing an

ocean with traveling far from home. Again, I infer that Shelby was bounding her experience by her start and end location (von Glasersfeld, 1981), not necessarily the duration of sitting on the plane before it took off or after it landed. Similar to Lennon in Excerpt 4.18, Shelby's conception of the duration of her experience seems to correlate with her movement through space (Piaget, 1969).

To quantify the duration of his experience, 4-year-old Cody also considered the distance he traveled as he flew in an airplane. During our initial interview, Cody and his family were packing to leave on a vacation to Arizona, a place to which he had previously been. During both interviews, I asked Cody to reflect on his experiences of traveling to Arizona. Excerpt 4.21 presents Cody's description of this travel both before leaving for Arizona (initial interview) and after arriving in Arizona (follow-up interview).

Excerpt 4.21 (August and September 2020): Cody's Description of Distance Traveled as he Conceived of the Duration of Traveling to Arizona

1. C (Cody): Yeah, but some time we're going to **fly to Arizona**!
2. A: When are you flying to Arizona?
3. C: Like, when we get to fly **on the airplane** then we will get to go to Arizona!
4. A: So, you fly to Arizona?
5. C: Yeah, but we need (to) drive there a bit and then we need to **fly there**.
6. A: Does it take longer to drive to the airport or to fly to Arizona?
7. C: Fly to **Arizona**!
8. A: Why does it take longer to fly to Arizona?
9. C: [hesitates for 4 seconds] Uh because um, uh, just because it's far away.
10. A: So, because the airport is closer, it doesn't take as long?

11. C: [shakes head] No.

Cody began by describing his accumulating activities of driving to the airport then flying to Arizona (Line 5). This comparison seemed to lead to his consideration of distance traveled, with implied spatial boundaries (von Glasersfeld, 1981) similar to Shelby in Excerpt 4.20. As he re-presented both activities of his experience, he seemed to quantify flying to Arizona as longer than driving to the airport because it was farther away (Line 7-9). During his follow-up interview, Cody was still on his vacation in Arizona. As he excitedly told me about his trip, I was curious as to how he might reflect on the duration of his flight having a more recent experience to reflect on (I continue Excerpt 4.21 below).

12. A: How long did it take you to get to Arizona?

13. C: A **long time**!

14. A: Can you tell me about that long time? How did you know it was a long time?

15. C: [spreads his arms as wide as he could]

16. A: That long!

17. C: Yeah, because the airplane **is fast** and then it took a **very fast time**!

18. A: What's the difference between a long time and a fast time?

19. C: Uh, when they are together, then it goes [hesitates briefly] um, 'til a long time **and a fast time**.

20. A: So, it was both long and fast?

21. C: [nods affirmingly]

Once again, Cody quantified the duration of his travel on an airplane as "a long time" (Line 13) and seemed to explain the duration through his gesturing (Hornbein, 2015) of a big distance (Line 15). I infer that this physical representation was reflective of Cody's consideration

of a great length, or distance, traveled. Cody then went on to describe the rapidity of the airplane, stating that the plane was fast and took a fast time (Line 17). Similar to my contention from Mick's description of baking muffins (Excerpt 4.10), I would not consider Cody's reflection on the rapidity of the plane to be his consideration of duration as a result of the rapidity of his actions—namely because he seemed to be reflecting on the actions of the airplane, of which he has no agency.

To better understand how Cody conceived of the duration of his travel, I asked how a long time was different than a fast time, as Cody seemed to coordinate a long time with a fast time (Line 17). This, I conjecture, was reflective of Cody's coordination between the space of his experience and the duration, similar to Lennon (Excerpt 4.18) and Shelby (Excerpts 4.19 and 4.20). When Cody stated, "when they are together" (Line 19) he seemed to explicitly link duration and rapidity as two independent attributes of his experience, rather than conceiving of the impact of rapidity on duration. He went on, stating "then it goes… 'til a long time and a fast time" (Line 19). I infer the "it" to which Cody was referring was the airplane going from his home state to Arizona, which again seemed to focus Cody's conception on the distance traveled by the airplane through his experience.

Summary of Duration as a Consideration of a Gross Quantity

The data presented in this section displayed a range of examples to demonstrate the participants' interiorization of duration as consideration of a gross quantity. My analysis revealed three different conceptions of duration as a consideration of a gross quantity: (a) qualitative (non-enumerated) and quantitative magnitude; (b) relative size; and (c) distance traveled. All seven participants seemed to present evidence of considering the duration of at least one experience through a consideration of a magnitude. The most common consideration of magnitude in the

data was conceiving of duration through a re-presentation of a qualitative magnitude, or a global comparison (Piaget, 1965), such as Tanner's description of "all" the marbles in Excerpt 4.12. I only found evidence of consideration of a quantitative magnitude by the 6-year-old participants, which may be reflective of a different way of reasoning, such as the construction of a unit of one (Steffe & von Glasersfeld, 1985; Ulrich, 2015).

When conceiving of the duration of their experience, all seven participants seemed to reflect on relative size in at least one of their experiences (e.g., the duration of building a big versus small LEGO, the duration of running in a big versus tiny circle). Each occurrence that I found of duration as a consideration of relative size was described in conjunction with other attributes of the participants' experiences, such as their accumulating activities or the magnitude of specified objects. I conjecture this may be because when conceiving of relative size, the participants were perceptually comparing (Piaget, 1965) multiple aspects of their experiences, creating more detailed descriptions.

Consideration of distance traveled was the least common reflection on gross quantity that I found during my analysis. Twice during my interviews, I anticipated that Lennon would consider his distance traveled as he described his experiences (running, Excerpt 4.17 and to and from school, Excerpt 4.18). However, each time, Lennon seemed to attend to other attributes of his experience as he re-presented the duration of his actions. Only two participants, Shelby (age 6) and Cody (age 4), seemed to conceive of duration through distance traveled—and both did so as they re-presented far distances on an airplane, which they conceived of as being a long duration. Additionally, Shelby reflected on distance as she re-presented the long duration of her walk to the park. This is perhaps similar to Reisman's (1971) assertion that larger units of time

are easier for children to conceptualize (as I noted in Chapter III). Next, I present evidence of my participants' conceptions of duration as a result of exertion.

Duration as a Result of Exertion

In this section, I use Wolcott's (1994) *description*, *analysis*, and *interpretation* to demonstrate how a child might conceive of duration as a result of exertion. I characterize *exertion* as how the participants explicitly attended to *their* actions and perceived those actions as impacting the duration of their experiences. Through my analysis, I distinguished two types of exertion: effort and rapidity (Piaget, 1969).

By effort, I mean how the participants perceived the ease (or challenge) of the activities they completed during their experiences. Some participants clearly stated that certain activities were "easy" or "hard," such as Shelby (Excerpt 4.1) stating that vacuuming her mom's room would be "a hard time." Other instances, the participants may not have explicitly stated that an activity was easy or hard, but rather made statements where the effort could be inferred. For example, as Easton described building couches out of LEGO, she stated "I *just* put a peach back on." I infer that the word "just" indicated ease in the perceived effort of her actions. I, therefore, characterized Easton's conception of the duration of building a LEGO couch as reflection on her exertion.

I characterize rapidity as how quickly or slowly the participants perceived *their* actions to occur during a given experience (Piaget, 1969). This is not necessarily the calculated speed of their actions, but rather their reflection on how swiftly they performed their actions. The coordination between rapidity and duration one of the most noted attribute of duration from past literature on children's reasoning of time (as detailed in Chapter II); and is fundamental to

Einstein's notion of space-time (Chapter I). Therefore, it was not surprising to me that all seven of the participants described the impact of the rapidity of their actions on the duration of their experiences. I now present some of those descriptions.

Duration as the Result of the Rapidity of Actions

When reflecting on duration as a result of rapidity, the participants seemed to reflect on how the swiftness of their actions might have impacted the duration of their experiences. I characterize this differently than reflecting on the speed of external attributes of experience, such as Cody's description of the airplane flying fast in Excerpt 4.21. In Cody's experience, he had no agency in the rapidity of his travel. The speed of the airplane was outside of his control, and it seemed that he reflected more on the quick speed of the plane rather than considering how it impacted the duration of his experience. Four-year-old Kyla reflected on the relationship between the rapidity of her actions and duration of her experience as she reflected on spinning in her chair.

Kyla started preschool remotely in August 2020. She attended her classes from home. During her initial interview, Kyla said that she didn't like when she had to sit at school, "Because you just get to sit and sit and sit." A few weeks later, I asked her how remote learning was going during her follow-up interview. Kyla told me that during school, she had a special "big, nice blue soft chair" that she got to sit in for school time. She excitedly talked about how the chair spun at a medium speed when she sat in it. I was curious how Kyla conceived of the rapidity of her spin on the duration of her experience, as Excerpt 4.22 shows.

Excerpt 4.22 (September 2020): Kyla's Consideration of the Rapidity of Spinning in a Chair on the Duration of her Experience

1. A: If you spin it faster, does it take longer to go around or shorter?

2. K (Kyla): Long.

3. A: How come?

4. K: Because you **spin** it.

5. A: What if you spin really slow, would that take a long time or a little time?

6. K: **A little** time.

7. A: How come?

8. K: Because then it goes slow.

9. A: So slow means a little time?

10. K: Yeah [nods affirmingly].

I inferred that Kyla reflected on the rapidity of spinning in her chair and correlated the rapidity with the duration in an intuitive way (Piaget, 1969). Kyla's response that spinning faster would take longer (Line 1-2) seemed to reflect her sensorimotor perception (as I described in Chapter II), with no apparent re-presentation of her past experiences of spinning faster (or slower, Line 5-10). When I asked how she knew the durations of spinning faster or slower, Kyla reflected on her activities (spinning, Line 4) and the rapidity itself (goes slow, Line 8).

Kyla reasoned that spinning slower took a small amount of time because "*it* goes slow" (Line 8), but it was unclear if "it" referred to the duration or the rapidity. If Kyla were reflecting on duration, it would seem that she was conceiving of duration a variable quality of her experiences. In other words, if duration were variable to Kyla, it would seem that she is yet to construct the uniformity of time (as I described in Chapter II). If she was reflecting on rapidity when she stated, "*it* goes slow," then I surmise that she demonstrated duration through her sensorimotor actions where the slow rapidity caused a shorter duration because it is slower.

Either way, it would seem that based on her description, Kyla has an intuitive conception of time (Piaget, 1969).

Unlike Kyla, 5-year-old Mick described an inverse relationship between the rapidity of her actions and the duration of her experience. During her initial interview, I asked Mick if she could think of something that took a short amount of time to do. Excerpt 4.23 presents her consideration of the rapidity of her actions on the duration of her "short" experience.

Excerpt 4.23 (August 2020): Mick's Reflection on the Rapidity of her Actions on the Duration of her Experience

1. A: Can you think of something that takes a short time to do?
2. M (Mick): Mmm [hesitates for 7 seconds and looks around] play with toys?
3. A: Play with toys. How do you know that just takes a little time?
4. M: Because you have to get them **really fast**.
5. A: So, when you play you get them fast? Can you make playing take longer?
6. M: Yeah.
7. A: What would you do to make it take longer?
8. M: Um, walk really **slow**.

To me, Mick seemed to re-present the rapidity of her actions as she conceived of the duration of playing with toys (Line 4). She did not seem to reflect on any other attribute of her experience to explain why playing with toys took a short amount of time (e.g., her accumulating activities of playing or the number of toys she played with). Her description, "you have to get them really fast" (Line 4) seemed to reflect the start of her activity (getting toys) and the swiftness that she got them, "really fast." I was curious if she would re-present the activity of

playing with the toys if the duration were longer, hence my question (Line 5-7). However, she seemed to continue to reflect on her rapidity of walking to get the toys (Line 8). Perhaps the experience Mick was re-presenting, was getting her toys to play with, not necessarily playing with her toys. This might be the reason that Mick conceived of playing with her toys as a short duration. Upon reflection, I should have asked her to elaborate on her experience to better identify what experiences she was actually re-presenting. However, this clarification does not negate Mick's description of an inverse relationship between the rapidity of her actions and the duration of her experience.

Similar to Mick, Shelby and Easton both reflected on the rapidity of their actions as they reflected on the duration of a short experience. Shelby and Easton each described brushing their teeth quickly during their initial interviews. I begin with Shelby's description. Shelby had explained that when her mom brushed her hair and put it in a ponytail it took a short amount of time (Excerpt 4.7). I asked if she could think of something else that took about the same amount of time, Excerpt 4.24 shows her response.

Excerpt 4.24 (August 2020): Shelby's Internalization of how the Rapidity of Brushing her Teeth Impacts the Duration of her Experience

1. S (Shelby): [hesitates briefly] Umm, brushing teeth because I, uh, I brush it **really quick**.
2. A: Oh, so when you're brushing your teeth, you said you brush it really quick—does how fast you brush it change how long it takes?
3. S: [hesitates for 6 seconds] Mmm, I don't know.

I inferred that Shelby seemed to reflect on how the rapidity of her actions (brushing quick, Line 1) caused a short duration, comparable to her hair brushing experience (Excerpt 4.7).

When I asked about the general relationship between these two attributes (rapidity and duration), Shelby hesitated for an extended time (Line 3). I conjecture that this hesitation indicated her consideration of this relationship; however, she seemed unable to verbalize this relationship. This may be because she was unaware of the inverse relationship between duration and rapidity (Piaget, 1969) or because she was not able to communicate her understanding to me. I now turn to Easton's description of her teeth brushing experience, and her reflection on how the rapidity of her actions might have impacted the duration of her experience in Excerpt 4.25.

Excerpt 4.25 (August 2020): Easton's Reflection of the Rapidity of her Actions on the Duration of Brushing her Teeth

1. A: Can you think of something that takes a short time?
2. E (Easton): Um, brushing your teeth.
3. A: How do you know it takes a short time?
4. E: Because you really, you **want** to do it quickly.
5. A: Oh, so if you brushed your teeth slower, would it take the same amount of time?
6. E: Mm, pretty much.
7. A: It would take the same amount of time?
8. E: Yeah.

I conjecture that Easton reflected on the rapidity of brushing her teeth as she conceived of the duration of her experience. To me, her emphasis of the word "want" (Line 4) indicated her perceived agency of the rapidity of her actions. Specifically, that she had the ability to change the rapidity of her actions if she so chose. I was curious how Easton understood this agency impacting the duration of her experience, thus I asked her about the durational impact of

brushing her teeth slower (Line 5)—similar to my follow-up question from Shelby's description in Excerpt 4.24.

When Shelby described the rapidity of brushing her teeth, her conception of how this rapidity impacted the duration of her experience seemed bound within her experience itself. She seemed unable to verbalize her anticipation of how the rapidity might impact potential experiences. Easton, on the other hand, anticipated that brushing her teeth slower would take the same amount of time (Line 5-8). Similar to my reflection from my conversation with Mick in Excerpt 4.23, I should have taken the opportunity to ask Easton to elaborate on how she knew that her experience would be the same amount of time if she brushed her teeth slower. However, this elaboration does not discredit the difference between Easton and Shelby's description of how the rapidity of their actions might impact the duration of their experiences—namely that Shelby was not sure if the duration would change, while Easton stated that the duration would be the same. Next, I continue my participants' reflection on the inverse relationship between the rapidity of their actions and the duration of their experiences by presenting a description from the third 6-year-old participant, Lennon.

During his follow-up interview, Lennon explained how long it took him to get to and from school, described in Excerpt 4.18. He began by describing walking versus riding his bike to and from school, then went on to explain that the trip to school was downhill while the trip home was uphill. There was an implied bounding of Lennon's experience (von Glasersfeld, 1981) between his house at the top of the hill and the school at the bottom. As Lennon re-presented his experiences, he seemed to attend to the rapidity of his actions as well as the effort of his travel (which will be more thoroughly discussed in the next section). To clarify Lennon's consideration

of the impact of rapidity on duration, I asked him about how the rapidity of his travel might impact the duration of his experience, as shown in Excerpt 4.26.

Excerpt 4.26 (September 2020): Lennon's Consideration of the Rapidity of Riding his Bike as he Conceived of the Duration of Traveling To and From School

1. A: So, pedaling your bike makes your speed go faster, does that make your time go faster?
2. L (Lennon): Uh, no [hesitates briefly] it doesn't.
3. A: How come?
4. L: Because, I don't know!
5. A: When you go home from school, you have to go uphill, tell me about going uphill.
6. L: [hesitates briefly] Um, uphill is different, you're kind of, you're going on the same hill, but it's uh, it's different cause you're **going up** and then you ride your bike **down** then it's faster if you ride your bike up it might be slower.
7. A: What is slower, the speed or your time?
8. L: Speed.
9. A: How does that affect how long it takes?
10. L: I don't know [shrugs shoulders].

Lennon seemed to reflect on the rapidity of riding his bike uphill, which he described as being slower, versus downhill, which he described as being faster. However, when asked to generalize how his rapidity might relate to the duration of his experiences, Lennon seemed unable to explain his reasoning (Line 4, 10). I infer this might indicate that Lennon's interiorization of such durational generalizations seemed to be developing, similar to Shelby in Excerpt 4.24. To prompt his thinking, I re-centered Lennon on the specific duration of getting to and from school (Excerpt 4.26 continues below).

11. A: Does it take longer to go to school or to get back home from school?

12. L: But—it takes, it takes longer to **go home** from school.

13. A: Why do you think that is?

14. L: Cause you might not be right th—cause it's **uphill**. Uphill can sometimes, it can be slower (and) can make, like, the tires **slow**.

15. A: Why does it make the tires slow?

16. L: I don't know [shakes his head and laughs].

To me, Lennon seemed clear in his description of the duration that coming home from school took longer than the duration of going to school from his house. Once again, I infer that he reflected on the effort of traveling which caused the rapidity of his actions. Here, he focused on the uphill travel of getting home, which caused his tires to move slower (Line 14).

Lennon's use of the word "sometimes" (Line 14) seemed significant. When considering the duration, the conception that time is *sometimes* (i.e., sometimes fast, sometimes short, etc.) might be reflective of the concept of the uniformity of time (described in Chapter II). Additionally, when Lennon stated, "*it* can be slower," (Line 14) it was unclear if "it" was referring to the duration of his uphill travel or the rapidity of his travel. While rapidity can be variable, duration cannot—thus, what Lennon was reflecting on during this excerpt may be telling of his durational reasoning and was something that I should have clarified during my interview. I continue my analysis of Lennon's description of riding his bike as I turn from duration as a result of the rapidity of actions to duration as a result of effort.

Duration as the Result of Effort

Through my analysis, I found that only the 6-year-olds seemed to conceive of duration as a result of their effort—and only during their follow-up interviews. This is not to say that younger children might never consider this attribute as impacting the duration of their experiences, just that in my data, there were no such occasions. Perhaps reflection on effort is indicative of a maturation or development in durational reasoning.

In Excerpt 4.26, I presented a conversation I had with Lennon about his experience riding his bike to and from school, specifically how the rapidity of his actions seemed to impact the duration of his experience. Prior Excerpt 4.26, Lennon had more generally described his experience of traveling to and from school, presented in Excerpt 4.27.

Excerpt 4.27 (September 2020): Lennon's Description of Traveling To and From School on his Bike (Continuation of Excerpt 4.18, Prior Conversation to Excerpt 4.26)

1. A: When you walk from your house to the school, how long does that take?
2. L (Lennon): Probably, 5 minutes.
3. A: How do you know it takes 5 minutes?
4. L: Because I usually **ride my bike**.
5. A: Oh!
6. L: Actually, cause my mom wouldn't let me because I actually go to school, so I don't ride my bike when I go to school just to play.
7. A: When you do ride your bike to school, can you get to school faster than if you walk or slower?
8. L: Uh, faster! If you ride **faster**!
9. A: Why?
10. L: Cause, **it has wheels** and wheels can go **fast**!

While Lennon described the rapidity of his actions (Line 8-10), the rapidity did not seem to pertain to the duration of his experience but rather the ease of his efforts. I conjecture that his last statement, "Cause, it has wheels and wheels can go fast" (Line 10) illustrated his consideration of the ease of his efforts. Specifically, I infer that Lennon implied that *because* his bike has wheels it goes fast, which then minimized his efforts in his experience. From this, I tried to bring the conversation back to the duration of Lennon's experience:

11. A: Okay! So, if you were to walk to school, how long do you think that would take you?
12. L: Probably [breathes heavily and blows out air] 10 minutes.
13. A: So, you think it would take 5 minutes to ride your bike and 10 minutes to walk to school?
14. L: Yeah.

Lennon seemed to clearly distinguish his actions in the duration of his experiences. When he deeply exhaled (Line 12), I infer that he seemed to be re-presenting his experiences as he reflected on the duration. From this consideration, Lennon seemed to support the notion that the act of walking versus riding his bike changed the duration of his experience (Line 13-14). This, to me, seemed to highlight Lennon's perceived effort of getting to and from school. I continue Excerpt 4.27 below.

15. A: Yeah?
16. L: Maybe [shrugs shoulders].
17. A: How far do you think you live from the school?
18. L: Probably either 5 or 10 minutes.
19. A: That's how far?

20. L: Yeah! I go **down** to school.

21. A: Do you think it makes a difference because when you go to school, it's downhill?

22. L: Yeah [nods affirmingly] That is, that question goes **with my bike!**

23. A: Explain that.

24. L: Um, downhill is if you're, like, riding **super-fast** and there's a, a really **long** downhill, um, then that can make you go **even faster**. The downhill and the tires, if you pedal really, really fast. That can, causes you going **really fast**, it can cause [sic] fast.

I found Lennon's last two statements (Line 22, 24) significant in his conception of duration as the result of his effort. As Lennon described going to school, he connected his travel downhill with going faster on his bike. Lennon seemed to focus specifically on a correlation between going downhill and riding his bike, through his explanation of the rapidity of his travel (i.e., he could go quickly downhill because of his bike). Once again, to me, Lennon's description of the rapidity seemed more suggestive of his reflection on how traveling downhill and being on a bike made his experience easier, specifically when he said, "The downhill and the tires" (Line 24). I infer that Lennon perceived riding "super-fast" as an easier activity because of "the downhill and the tires," versus, say, when he had to walk uphill. When I asked how "the downhill and the tires" related to the duration of his experience, Lennon was not sure, as shown:

25. A: So, pedaling your bike makes your speed go faster, does that make your time go faster?

26. L: Uh, no [hesitates for 4 seconds] it doesn't.

27. A: How come?

28. L: Because, I don't know!

From this description, I infer that Lennon conceived of the duration of getting to and from school as a result of his effort. Specifically, there seemed to be two distinct aspects of his perceived effort that Lennon attended to as he reflected on the duration of his traveling experience. One aspect seemed to be his consideration of the effort involved in his travel downhill versus uphill to and from school—namely that downhill was easier. I infer that the second aspect Lennon seemed to consider was the effort that was exerted by the tires while he was riding his bike versus when he was walking to school. Both aspects, to me, seemed notable during Lennon's reflection on the duration of his experience. While Lennon's perceived effort involved other aspects of his experience, his bike and the hill, Easton seemed to reflect on the efforts of her own actions as she described her LEGO building experience.

As previously presented in Excerpt 4.9, Easton described her experience of building LEGO couches during her follow-up interview. When describing building the larger couch, Easton seemed to reflect on her accumulating activities and the relative size of her couch. When describing the smaller couch, Easton instead seemed to consider her perceived competence with LEGO building as she recalled her experience, as Excerpt 4.28 shows.

Excerpt 4.28 (September 2020): Easton's Reflection on her Perceived Effort of Building a Small LEGO Couch (Continuation of Excerpt 4.9)

1. A: Okay! Tell me a little bit about the little couch.

2. E (Easton): Um, the little couch has, like, a pattern of tiles, two green and one turquoise or two turquoises and one green. Like, like, minty turquoise, like, light really minty turquoise.

3. A: Pretty colors. And you said that one didn't take you as long to make?

4. E: [shakes head] No, and I **just put a peach back on** and then I just moved it to the top and then I made, and the arms are completely different from each other.

5. A: Nice, what are the arms?

6. E: One arm is, like, like, like, a little slide that's green, but really thin. They're on the same side. And the other is, like, a little awning.

7. A: Cool! What a good idea. And it took you less time to build the little one?

8. E: Yeah [nods affirmingly].

9. A: How do you know it took you less time to build that one?

10. E: Um, because I didn't really **have to,** like, **think** about it. I, like, **knew** I was gunna do a couch.

As previously stated, I inferred that Easton's statement, "I *just* put a peach back on" (Line 4), indicated an ease in the effort she perceived of her experience—just meaning simply, or without effort—which she again stated as she moved the pieces (Line 4). This was the first activity mentioned in her little couch building experience, perhaps indicating the starting boundary of her experience (von Glasersfeld, 1981). At the end of the excerpt, Easton seemed to clearly state that the small couch took her less time to build because she "didn't really have to think about it" (Line 10). Nowhere in Easton's description did she seem to consider the size of the couch (small), nor the number of pieces used to build it. Rather, she seemed to reflect on her aptitude in building the couch as an explanation for her quantifying the experience as a short amount of time. Her last sentence, "I, like, **knew** I was gunna do a couch" (Line 10) seemed like an indirect comparison between her activities of building the two couches. Figure 10 shows this comparison.

When Easton described building the bigger couch in Excerpt 4.9, she explained that she started by building a makeup stand then changed her mind and decided to build a couch. Easton explained that this change in her thinking caused the duration of her experience to be longer—although her description of the experience seemed to integrate her accumulating activities with

the magnitude of LEGO pieces and relative size of the couch. In the current excerpt, Easton seemed to know what she was going to build, and then she built it. To me, this ease of effort seemed to explain her conception of building the small couch as a short duration.

Figure 10 is a continuation of Figure 7 and shows the difference in Easton's re-presentations of building the two couches. Specifically, the narrative figure illustrates how Easton's perceived ease of building the small couch created a much shorter, more focused description than her description of the accumulating activities and relative size (gross quantity) of building the larger couch. I now turn to my final example of the 6-year-old participants' reflection of duration as a result of their perceived effort, with Shelby's coordination of her reflection on magnitude and her perceived effort.

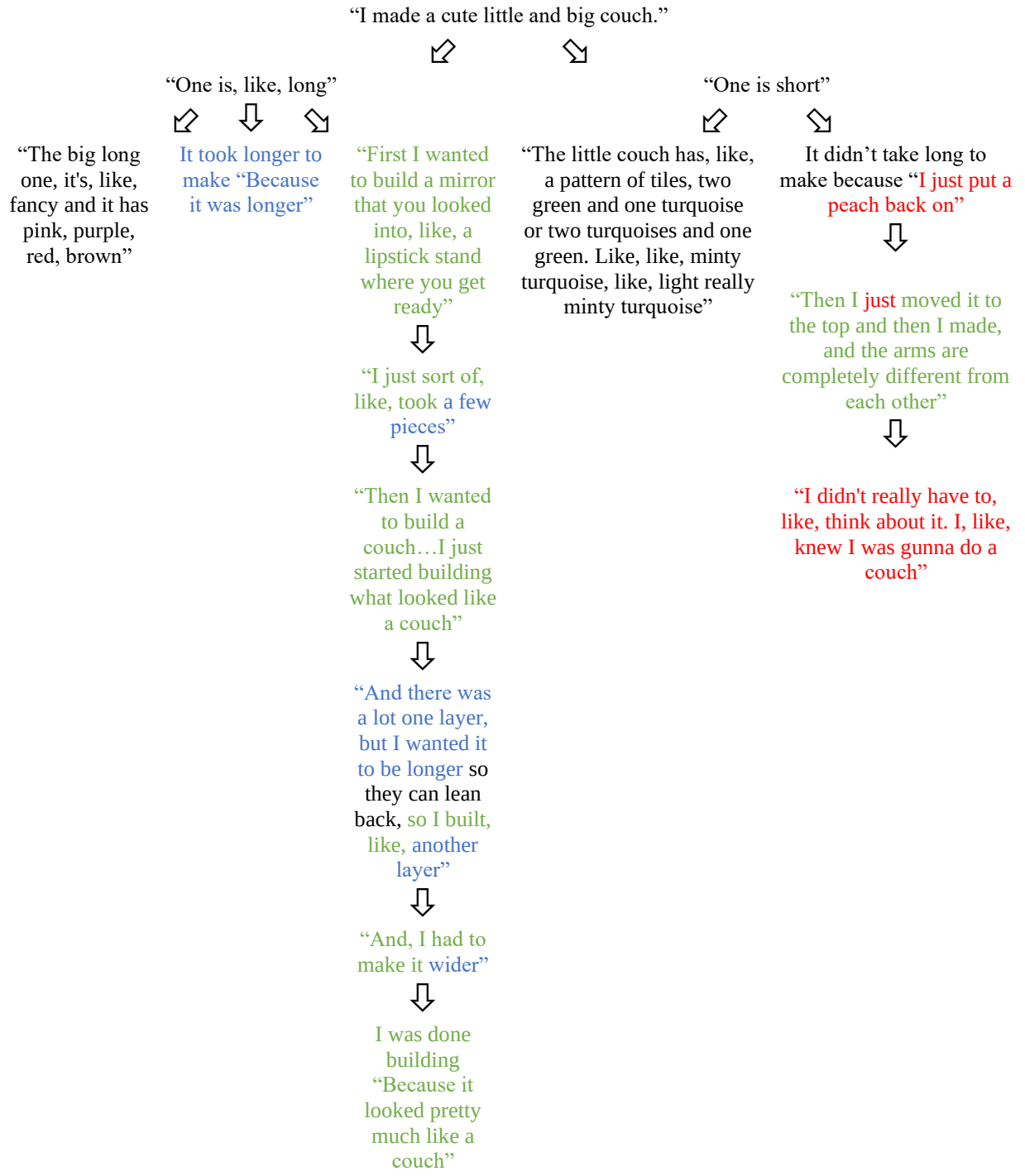
From my illustration of my data analysis at the beginning of this chapter, I presented a conversation I had with Shelby about the duration of her vacuuming experiences (Excerpt 4.1). Shelby had explained that her mother's room would be hard work that would take a long time. Prior to this, Shelby had been describing her experience of organizing her Pokémon cards, as shown in Excerpt 4.29.

Excerpt 4.29 (September 2020): Shelby's Description of her Perceived Effort of Organizing Pokémon Cards

1. A: When you're organizing your cards, you said it takes a long time. Why is it taking you a long time?
2. S (Shelby): Because, um, if you have a lot of cards that you can put in before, and then you have **new cards** and makes a harder **time**.
3. A: What's the difference between a harder time and a longer time?
4. S: Harder means, like, **more stuff** to put in.

Figure 10

Continued Narrative Visualization (Segel & Heer, 2010) of Easton's LEGO Building Description



Shelby's last statement, that harder means "more stuff" (Line 4), suggested to me that she was correlating her effort with the magnitude of cards, but not necessarily with the duration of her experience. This seemed reasonable to me as I considered her account. Unlike vacuuming, Excerpt 4.1, which involves more strenuous physical exertion, placing cards in a binder seemed easier. However, when reflecting on vacuuming a single room versus putting dozens of cards into a binder, the ease of activity seemed to change for Shelby. Here, I surmise, Shelby conceived of the larger quantity of cards creating a harder experience, which caused a longer duration. In other words, her efforts were a result of her endurance to put away a large number of cards, rather than exert herself physically as she vacuumed one room. Shelby's consideration of the magnitude of cards did not seem unitized in such a way that she could iterate the activities (organizing single cards, vacuuming single rooms) to quantify the duration of her experiences (Steffe, 1991; von Glasersfeld & Steffe, 1985). When I asked Shelby to clarify her perceived relationship between her efforts and the duration of her experience, she responded:

5. A: Does that mean it's gunna be longer if it's harder?
6. S: Yeah.
7. A: Is there a way that you could make it easier?
8. S: [hesitates for 3 seconds] Probably? If you have cards that you didn't put in, and you need to **put those in first** then you need to put another one.
9. A: And that would make it easier?
10. S: Yeah.

To assess my conjecture that Shelby was equating effort to the magnitude of the cards, I asked specifically about her efforts, and how she might make her experience easier (Line 7). First, Shelby hesitated, and then responded "Probably?" with an upward inflection that I interpret

as demonstrating more hesitation in her reflection. This, I believe, was evidence of her attempt to anticipate potential activities and consider her efforts of these activities. Shelby's consideration of putting some cards in first (Line 8) seemed to indicate a consideration of breaking her activities up across multiple experiences to ease the effort of her actions during each experience. In other words, doing some work today and the rest tomorrow, or spreading the work out over time. I infer that, to Shelby, this would ease her perceived effort, causing her experience to be easier. However, I was not clear on how she conceived of the duration of this "easier" experience, so I went on:

11. A: Do you think if you made it easier, it would make it take longer or shorter?

12. S: Probably shorter.

13. A: How come?

14. S: [hesitates briefly] Like, if you, if **yesterday** if you have cards and you put those in because you know you're going to get a Pokémon on the other day, you need to put away the cards.

15. A: Hmm. So, I'm thinking about this idea that you're talking about if you started something yesterday and then today you kept working on it, you think that would make it take less time today?

16. S: Uhh [hesitates for 5 seconds] probably?

17. A: Why?

18. S: Mmm [hesitates for 5 seconds] uh, cause if you have, like, **a lot of Pokémon cards** on the floor, then, um, it would make it harder time. But if you have a **little bit**, just, like, one card or two, it will **make it easier**.

19. A: And easier means less time?

20. S: Yeah [nods affirmingly].

To me, as Shelby explained why the duration of her experience would be shorter if her perceived effort was easier (Line 14), she seemed to be attending to the accumulation of activities she might experience if she were putting her cards away over the course of two days ("yesterday" and "the other day" (Line 14)). She still seemed hesitant in her anticipated duration of this experience (Line 16, 18), and returned to a consideration of the magnitude of the cards (Line 18) as a justification for her efforts—a lot of cards is a harder time, less cards is an easier time. Once again, I tried to bring Shelby back to the duration of her experience by asking if she conceived of her experience being easier if the event took place over a shorter amount of time (Line 19), to which she affirmed. I infer that Shelby correlated her ease of effort with the duration of an experience (e.g., easier is less duration), similar to the relationship between velocity and duration (e.g., quicker is less duration; Piaget, 1969).

Summary of Duration as a Result of Exertion

The data presented in this final section are examples of my participants' reflections on duration as a result of exertion. Specifically, the data demonstrated how the participants re-presented *their* effort and *their* rapidity as they described their experiences, to conceive of duration as a measurable attribute of those experiences.

Through my analysis, I found that only the 6-year-old participants seemed to consider their effort when describing their experiences. Again, this is not to say that the 4- and 5-year-olds were not re-presenting the effort involved in their activities, but they did not provide any specific indication of such considerations. As Shelby, Easton, and Lennon reflected on their effort, they seemed to associate the ease (or challenge) of their experiences with the magnitude of work (e.g., there were a lot of cards to put away) or their competence (e.g., I just knew what I wanted to do).

The participants seemed to consider how this effort directly impacted the duration of their experiences.

All seven participants seemed to reflect on rapidity at some point during their interviews, however as shown by Mick and Cody (Excerpts 4.10 and 4.21, respectively), not all described their agency in the rapidity of their actions when conceiving of the duration of their experience. The relationship between rapidity and duration is a common theme in past research on children's durational reasoning, as I described in Chapter II (Piaget, 1969). Specifically, when a child conceives of an experience as necessarily being a shorter duration because they completed the activities that filled it faster (or vice versa). From my analysis, neither 4-year-old participant seemed to re-present this inverse relationship through their accounts. This seemed consistent with past research, namely young children's intuitive conception of time (Piaget, 1969). Among the 5- and 6-year-olds, I found no apparent progression in how the participants seemed to conceive of this relationship. Children from both age groups did and did not seem to conceive of this relationship as they re-presented their experiences.

To Summarize

In this chapter, I presented data of 4-, 5-, and 6-year-old's descriptions of duration as an attribute of their lived experiences. My exploration revealed several common attributes that the participants seemed to reflect on as they quantified the durations of their experiences. I articulated three distinct themes from these attributes: (a) duration as an accumulation of activities completed; (b) duration as a consideration of a gross quantity; and (c) duration as a result of exertion.

I conjecture that when my participants conceived of duration as an accumulation of activities completed, they seemed to attend to the distinct activities that comprise a given

experience. The participants bound their experiences (von Glasersfeld, 1981) through their re-presentation of either a single, repeated activity or a collection of unique activities. They then conceived of the duration of their overall experience based on their reflection of the accumulated activities.

When my participants conceived of duration as a consideration of gross quantity, I infer that they reflected on the amount of a specific attribute of their experience to quantify the duration of the experience. The participants attended to a variety of gross quantities, including qualitative (non-enumerated) and quantitative magnitude of a given feature, the relative size of a specific attribute, or the distance traveled during their experiences. These gross quantities became the objects on which the participants reflected to spontaneously compare the duration of their experience (Piaget, 1965).

I found that when the seven participants conceived of duration as a result of exertion, they seemed to focus on how *their* actions might affect the duration of their experiences. The participants reflected on their perceived effort during the activities of their experiences and the rapidity in which they were able to perform those activities. Then, they conceived the duration of their experiences as a result of those actions.

From the data analyzed, there was no clear progression across my participants' conceptions, nor could I make any claims about how a child might progress from one way of reasoning to another. The data presented is intended to serve as markers (Tzur, 2019) for possible ways that children might conceive of duration as an attribute of their lived experiences.

CHAPTER V

DISCUSSION

All we have to decide is what to do with the time that is given us. (Tolkien, 1954/1994, p. 50)

Earnest (2019) concluded his paper with a call to action:

> [T]oo often studies involving time in maths education are divorced from duration, the quantity that time measures…Research on children's time-related ideas ought to consider not just symbol systems for time but how such symbol systems speak to the duration as an ongoing experience, and therefore future studies ought to incorporate the experience of duration into the study design. (p. 25)

I began this study with an explicit focus of exploring how children re-present duration as an attribute of their lived experiences. My research focused on how 4-, 5-, and 6-year-olds conceived of duration as a measurable quantity of their lives, prior to formal, school-based instruction on time (CCSSI: Measurement and Data, 2020), when such symbol systems for time are introduced. Specifically, my research addressed the following question:

> When reflecting on past experiences, what conceptions do young children seem to have about the duration of those experiences, as indicated by their descriptions of duration?

I conducted two interviews with each of seven children from affluent, middle-class families in a large metropolitan area in the Midwest, United States. My initial interview with each participant in August 2020 followed a semi-structured series of questions designed to elicit their durational reasoning (shown in Table 2 of Chapter III). I completed follow-up interviews with each child in September 2020. In those follow-up interviews I addressed specific questions

arising from my reflexive notes (from the first interview) and explored their conceptions of the duration of their idiosyncratic experiences.

My analysis (Chapter IV) led to distinguishing three common themes across my participants' conceptions: (a) duration as an accumulation of activities completed; (b) duration as a consideration of a gross quantity; and (c) duration as a result of exertion. While these themes were not mutually exclusive, they each indicated a different focus of the child's attention during their description. Figure 11 presents a thematic analysis (Nowell et al., 2017) of my inferences from all seven participants' conceptions of the duration of their lived experiences.

I attributed to my participants the first conception, duration as an accumulation of activities completed, when they re-presented the individual activities that created their overall experience, then used these activities to conceive of the duration of their experience. I found two manifestations of this conception, re-presenting a single activity that was completed multiple times and re-presenting a sequence of unique activities. An example of my participants conceiving of duration through repeated activities was 4-year-old Kyla's re-presentation of her teacher drawing an animal "wrong" then having to re-draw it "again" (Excerpt 4.3), causing Kyla to quantify the long duration of this experience. An example of my participants conceiving of duration through a sequence of unique activities was 6-year-old Easton's re-presentation of building a big LEGO couch (excerpt 4.9), which she quantified as a long duration because she first wanted to build a lipstick stand, then changed her mind and started building a couch, then progressively made it bigger by adding additional LEGO pieces. Through either conception, my participants seemed to directly attend to their accumulating activities as they conceived of the duration of their experiences.

Figure 11

Thematic Analysis (Nowell et al., 2017) of All Seven Participants' Descriptions Arranged by Age

Theme		Participant, Age	Description	Inferred Duration	Excerpt
Duration as an Accumulation of Activities Completed	Single Activity Repeated	Kyla, 4	"sometimes she gets it wrong and then she has to do it **again**"	Long	4.3
	Series of Unique Activities	Cody, 4	"sometimes she brushes my tongue **and my** teeth"	Little	4.6
	Single Activity Repeated	Mick, 5	"I had to do it **all over again**"	Long	4.4
	Series of Unique Activities	Tanner, 5	A: because you have the movie **and** the popcorn **and** the drink it takes longer? T: [nods affirmingly]	15 hours; Long	4.5
	Series of Unique Activities	Shelby, 6	"she brushes it…then she puts it up in a ponytail **and then it's done**"	Short	4.7
	Series of Unique Activities	Lennon, 6	"if you shove lots of things in your mouth and it'll be faster, but you'll have to swallow **a lot**"	Short	4.8
	Series of Unique Activities	Easton, 6	"then I just started building what looked like a couch and there was a lot one layer [sic], but I wanted it to be longer"	Long	4.9
Duration as a Consideration of a Gross Quantity	Qualitative Magnitude	Kyla, 4	"Because, I had to do **so many** dots"	Long	4.11
	Relative Size	Cody, 4	"that **big** LEGO ship that we have downstairs, that took longer"	Long	4.16
	Distance Traveled	Cody, 4	"we need (to) drive there a bit and then we need to **fly there**…because it's far away"	Long	4.21
	Qualitative Magnitude	Mick, 5	"there's a lot of **steam** in it"	Long	4.10
	Qualitative Magnitude	Tanner, 5	"we, like, set in **all the** marbles"	Long	4.12
	Quantitative Magnitude	Easton, 6	"maybe if it's, like, two **pages** or something"	2 minutes	4.13
	Quantitative Magnitude	Easton, 6	"I thought there was **another page,** but it was, like, none"	5-10 minutes	4.15
	Relative Size	Lennon, 6	"The bigger, **the longer**"	Long	4.17
	Distance Traveled	Shelby, 6	"we go somewhere a little bit **far**"	Long	4.19
	Distance Traveled	Shelby, 6	"I opened the window and it was, like, **far, far**"	Long	4.20
Duration as a Result of Exertion	Rapidity of Actions	Kyla, 4	"Because then it goes slow"	Little	4.22
	Rapidity of Actions	Mick, 5	"you have to get them **really fast**"	Short	4.23
	Rapidity of Actions	Shelby, 6	"I brush it **really quick**"	Short	4.24
	Perceived Efforts	Shelby, 6	"Harder means, like, **more stuff** to put in…and you need to **put those in first** then you need to put another one"	Long	4.29
	Rapidity of Actions	Easton, 6	"you **want** to do it quickly"	Short	4.25
	Perceived Efforts	Easton, 6	"I just moved it to the top… I didn't really have to, like, think about it. I, like, knew I was gunna do a couch"	Short	4.28
	Rapidity of Actions	Lennon, 6	"you ride your bike **down** then it's faster"	Short	4.26
	Perceived Efforts	Lennon, 6	"riding **super-fast** and there's a, a really **long** downhill"	Short	4.27

I attributed to my participants the second conception of duration as a consideration of a gross quantity (Piaget, 1965) when they reflected on a qualitative or quantitative magnitude, relative size, or distance of specific attributes of their experience. When considering magnitude, my participants seemed to reflect on the amount of some feature of their experience as they described the duration. For example, 5-year-old Tanner's said that it took a long time to complete his Marble Run "Because of how many marbles we setted [sic] in, we, like, set in all the marbles" (Excerpt 4.12). When considering relative size, my participants seemed to attend to the size of an object of their experience when describing the duration. This was illustrated in 4-year-old Cody's description of the varying duration of building big versus little LEGO spaceships with his dad (Excerpt 4.16). When considering distance during their experiences, my participants seemed to describe how far they traveled as they quantified the duration of their experience. For example, 6-year-old Shelby described the long duration of traveling "a little bit far" to get to the park (Excerpt 4.19). Accordingly, I ascribed this conception when the participants' reflection on the gross quantity seemed consequential to the specified duration (e.g., it took a long time *because* the park was far away).

I attributed to my participants the third conception, duration as a result of exertion, when they seemed to consider how *their* efforts, or the rapidity of *their* actions, affected the duration of their experience. Five-year-old Mick seemed to reflect on the rapidity of her actions (Excerpt 4.23) as she described getting her toys "really fast," which caused a short duration. She also reasoned about the reverse relationship that, by walking slower, she could make the duration of her experience longer. Six-year-old Lennon seemed to consider his efforts as he explained that riding his bike made the duration of his travel to school shorter than walking, because "the tires, if you pedal really, really fast" (Excerpt 4.27). He articulated that his travel to school would take

twice as long when walking than riding his bike. My attribution of this theme specifically addressed the participants' implied agency in the overall duration of their experiences. When looking across the data (shown in Figure 11), the majority of the descriptions that I inferred displayed my participants' conceptions of duration as a result of exertion came from the 6-year-olds.

The 6-year-olds also seemed to present more evidence of considering multiple conceptions during a single experience (as seen in Figure 11), for example Lennon's re-presentation of eating lunch by quickly shoving a lot of food in his mouth. Again, I am not suggesting that younger children do not consider multiple attributes of their experiences as they conceive of duration, but that my data found less evidence of this. Perhaps the presentation of such descriptions by the older participants is reflective of their ability to verbalize their experiences, and thus, my ability to make inferences into their conceptions.

What I found most notable about my participants' descriptions of duration through multiple attributes of their experience was that all of these occurrences included evidence of their conceiving of accumulating activities. This seems consistent with durational re-presentation, as the activities of an experience are reconstructed within the child's mind. It therefore seems plausible that conceiving of duration as an accumulation of activities might be foundational to other durational quantification—though further investigation would be necessary.

In this chapter, I discuss key contributions of this study to the field, its implications for teaching and learning, and its limitations. I conclude by presenting recommendations for future research on children's conceptions of duration as an attribute of their experiences. I begin with the key contributions of this study.

Key Contributions of this Study

By exploring how children reflected on and re-presented their lived experiences, I intended to begin mapping the terrain of children's conceptions of duration as an attribute of those experiences. As noted by Earnest (2019), the present breadth of research on children's conceptions of time has seemed to overlook implications of children's experiences on their construction of their durational reasoning (Burny et al., 2013; Earnest, 2017, 2018, 2019; Friedman & Laycock, 1989; Kamii & Russell, 2010, 2012; Levin, 1977, 1979; Long & Kamii, 2001; Metelerkamp, 2013; Piaget, 1969; Richie & Bickhard, 1988; Russell, 2008; Tillman & Barner, 2015). It is in this lacuna sense that this dissertation study seems significant for mathematics education researchers and preK-12 teachers. In this section, I elucidate three key implications of the study about how children might conceive of duration as an attribute of their experiences. First, I extend the research of Steffe and von Glasersfeld (1985; Steffe, 2010; von Glasersfeld, 1981) to consider how children might begin to construct durational units (Piaget, 1969) based on their bounded experiences. Second, I describe (and further conjecture) how a child's spontaneous correspondences (Piaget, 1965) might coordinate their concept of number (Piaget, 1965; Steffe & von Glasersfeld, 1985) with their conception of duration as a measurable attribute of their world (Earnest, 2018a). Finally, I support existing research on children's reflections on their actions as they conceive of potential relationships between duration and other attributes of their past experiences (Piaget, 1969).

Children's Development of Durational Units

> A unit is fundamentally a tool of measurement. (Ulrich, 2015, p. 3)

Von Glasersfeld's (1981) attention model (detailed in Chapter I and II) described the construction of unitary items through the focused and unfocused bounding of sensory-motor input. These unitary items result from brief moments of attention and could become the mental objects from which a child might abstract numerical units. This unitizing operation explicated children's construction of numerical units from their abstraction of the sensory-motor input from their experiences. Based on my analysis, I conjecture that children's development of durational units might follow a similar unitizing process as they conceive of duration as an accumulation of activities completed—thus extending the present theory of the unitizing operation (Steffe & von Glasersfeld, 1985; von Glasersfeld, 1981). Specifically, I purpose that my participants' re-presentations of successive sensorimotor activities may create a unit of duration when bound by their completion of a self-established task within a single experience.

As they re-presented the duration of their completed activities, the participants in this study first had to identify (for themselves) the boundaries of their experience. In other words, they had to reflect on the specific activities that began and concluded their experience. An example of this was 6-year-old Shelby's description of the short duration of her mother doing her hair (Excerpt 4.7)—which began when her mom brushed her hair and was done when her hair was in a ponytail. Because *these* were the specific activities that Shelby re-presented as she reflected on her experience, they bound—and thus defined—the duration of her experience. If, for example, Shelby had re-presented her mom washing and drying her hair before brushing it or braiding her hair after it was in the ponytail, she might have conceived of the duration of her experience differently. The boundaries a child had established, therefore, became fundamental to my participants' quantification of the duration of their experiences; their attention to the boundaries defined where they began and ended their durational measurement.

The re-presented activities between the defined boundaries, then, became the sensorimotor input from which my participants might abstract "a 'whole' or 'thing' or 'object'" (von Glasersfeld, 1981, p. 87 as presented in Chapter II) of duration. When conceiving of duration as an accumulation of activities completed, there is an inherent "unitariness–'things' that go together *because they are put together*" (Steffe, 2010, p. 31). An example of one such unitary conception was demonstrated during 5-year-old Tanner's description of the long duration of watching a movie (Excerpt 4.5). As Tanner re-presented his experience of watching a movie, he described the disparate activities of watching the movie, eating popcorn, and drinking. These three, separate sensorimotor items seemed to accumulate as Tanner conceived of the long duration of a movie. In other words, I conjecture that because Tanner re-presented these activities bound within a single experience, he seemed to both unitize (distinguish) and unite (put together) them to quantify the duration of his experience. When asked specifically about his reflection on the separate activities, Tanner affirmed that it was because of the accumulation that the duration of his experience was long.

As I noted in Chapter II, Steffe (1991) posited "Segmenting sensory experience into units is the result of a unitizing activity *prior* [emphasis added] to measuring or to counting" (p. 63). This suggests that the unitization of durational units I have presented might serve as a precursor to Long and Kamii's (2001) model of durational measurement. Before children can measure the duration of their experiences—which Long and Kamii explained requires conservation of speed, transitive reasoning, and unit iteration (as I presented in Chapters I and II)—they need to conceive of duration as a possible unit of measure. When unitizing their accumulating activities, I infer that my participants were conceiving of the duration of their experiences as a unit in such

a way that the durational unit might be comparable against the duration of other experiences (transitive reasoning).

Six-year-old Lennon seemed to reflect on this comparison as he organized the duration of his experiences against one another (Figure 6; Excerpt 4.8). When I presented Lennon with the durational organization task from Tillman and Barner's (2015) linguistic study, he seemed to noticeably consider how the duration of his different experiences might relate to one another, namely brushing his teeth and eating lunch. Before describing either experience, he conceived of the short duration of eating lunch as taking "not too much long(er)" than brushing his teeth. Thus, I conclude, Lennon re-presented the bounded activities of each experience, unitized the duration of each set of accumulating activities, then compared the two units of duration against one another. This, perhaps, is an early example of the development of qualitative durational measurement—with the unitization of duration being a central tool for such measurement (Ulrich, 2015). Next, I present my key contributions based on articulating children's conception of duration as a consideration of a gross quantity.

Children's Correspondences Between a Gross Quantity and Duration

> [T]he evaluation of 'lived' (inner) duration calls for a host of conscious or unconscious comparisons. (Piaget, 1969, p. 275)

As children develop their conception of number (described in Chapter II), they engage in qualitative (non-enumerated) and quantitative correspondences (Piaget, 1965). Piaget (1965) explained correspondences as comparisons between two quantities. Spontaneous correspondences, more specifically, are "situations in which the child compelled to find the correspondence of his own accord and to make what use of it he can" (Piaget, 1965, p.65). As the

participants in this dissertation re-presented the duration of their experiences, all seemed to correspond a gross quantity with the duration of their experience at some point. I conjecture that the spontaneous correspondences described by my participants demonstrated their global comparison between a tangible attribute of their experience and their perceived duration. This conjecture helps broaden the current theory of spontaneous correspondences (Piaget, 1965), by pointing to how a child may conceive of number with their early durational measurement.

When the participants considered a gross quantity while re-presenting their experience, they reflected on a non-enumerated quantity (e.g., lots, all, little, far). Unlike physical time (introduced in Chapter II; Piaget, 1969) or provoked correspondences (Piaget, 1965), my participants had to re-present the quantity for themselves, and did not have the actual items of their experience to count. Four-year-old Kyla, for example, described the long duration of her painting experience through her re-presentation of painting "so many dots…and so many lines" (Excerpt 4.11). Kyla's quantification of the dots and lines had inherent, accumulating activities of successively painting each dot and line. However, as she described her experience, Kyla seemed to attend to the magnitude of dots and lines, not her activity of creating them. She explicitly stated that it was *because* she had to paint so many dots and lines that the duration of her experience was long. In other words, I conclude that Kyla globally compared the large quantity of dots and lines against the long duration of her experience.

The other 4-year-old participant, Cody, seemed to reflect on his correspondence between a gross quantity and duration while he described his experience. As I previously noted, Cody seemed to attend to the relative size of his LEGO creations as he re-presented the duration of building with his father (Excerpt 4.16). Cody initially seemed to correspond "a lot [spreads arms wide] of pieces" with the long duration of his LEGO building experiences, similar to Kyla's

correspondence. Later, Cody's attention seemed to shift from the magnitude of LEGO pieces to the size of his builds. He stated that his dad had built a "big" rocket ship. When I asked if this also took a long time to build, Cody immediately responded, "No," then hesitated and switched his response to, "yeah." I infer that Cody's initial response, hesitation, and then durational adjustment demonstrated his in the moment, spontaneous correspondence. Specifically, his initial "No" would have meant that the "big" size created a short duration, which does not correspond quantitatively. Cody's adjustment demonstrated, to me, that he was actively and spontaneously correlating the "big" size and the long duration.

The 4- and 5-year-olds in this dissertation study described the gross quantities of their experiences qualitatively, through non-enumerated measures. Only 6-year-old participants (Shelby and Easton) named specific numerical values for various aspects of their experiences. Easton, for example, explained that a short duration would correspond to reading *two* pages of a book (Excerpt 4.13). I found Easton's use of a precise quantity ("two"), versus a general amount (some, a few, many) notable—as the correspondence she created between the number of pages and the duration of her experience could still have occurred with a non-enumerated quantity. Thus there seemed to be something more to her conceptualization of a short duration by her re-presentation of reading "two pages." I conjecture that the use of such quantitative magnitudes might correspond with the participants' development of their concept of number (detailed in Chapter II; Piaget, 1965; Steffe & von Glasersfeld, 1985; Ulrich, 2015) and, therefore, with their correspondence between a gross quantity and the duration of their experiences. I now present my key contributions of children's conception of duration as a result of exertion.

Children's Reflections on Their Actions and the Duration of Their Experiences

> [W]e are therefore entitled to conclude that…children judge time by the work done which, in turn, is judged by its difficulty and the effort it demands rather than its quantitative terms. (Piaget, 1969, p. 243-244)

Piaget's (1969) investigations of children's conceptions of time established a foundational developmental progression of children's durational reasoning (described in Chapter I and II). More specifically, his work was the only discussion that I could find in my extensive search of the literature on children's development of inner duration (psychological time; explained in Chapters I and II). Other studies have corroborated Piaget's general progression of children's intuitive to operational durational reasoning (Kamii & Russell, 2010; Long & Kamii, 2001; Russell, 2008). Data presented in Chapter IV on children's conception of duration as a result of exertion seem to substantiate some of Piaget's contentions about children's development of inner duration.

Piaget (1969) evaluated three temporal conceptions of inner duration: (a) "The rapidity of actions" (p. 231); (b) "Duration and tasks of varying difficulty" (p. 242); and (c) "Inactivity and interesting work" (p. 245). Through my conversations with this dissertation study participants, I gathered evidence of two of the conceptions Piaget investigated, rapidity of actions and perceived difficulty. This seemed significant to me because, while Piaget had begun his research with these conceptions in mind, my participants brought forth these conceptions upon reflecting on the duration of their past experiences, as when 5-year-old Mick explained that getting her toys "really fast" created a short duration (Excerpt 4.23). Mick's reflection of the rapidity of her actions seemed supported as she also anticipated the inverse—that getting her toys slower would

cause her experience to be longer. Piaget noted that "90 per cent [sic] of those between the age of 5;1 and 6;0 invariably believe that work done rapidly takes more time than work done slowly" (p. 235), which would make Mick uncharacteristic of his data. Four-year-old Kyla, on the other hand, reasoned that spinning slowly in her chair would take a little time because it was slower (Excerpt 4.22). In other words, Kyla seemed to reason that work done slowly takes less time, precisely what Piaget noted for his younger participants.

Piaget (1969) also found that his younger participants struggled to reason about the difficulty of their actions and the duration of their experiences. From my data analysis, I too found that the younger participants (the 4- and 5-year-olds) did not seem to bring the perceived ease of their activities into their re-presented duration of their experiences. I liken this to my previous conjecture about my 6-year-olds participants' development of number—perhaps there is a developmental consideration (e.g., maturation) to their perceived efforts during re-presentation. For example, I inferred that 6-year-old Easton's description of the short duration of building a small LEGO couch (Excerpt 4.28) demonstrated her perceived efforts. Multiple times during this description, Easton stated that she "just" had to do things, and then explained that she "didn't really have to**…**think about it." To me, Easton seemed cognizant of how the ease of her mental efforts created the short duration of her experience. This mental effort was different from Piaget's investigation, which involved physical exertion—perhaps expanding the existing research on children's conceptions of inner duration.

I found it notable that none of the participants in this dissertation study seemed to explicitly re-present their inactivity or engagement as they conceived of the durations of their experience. However, there might be a correlation between this conception and Lennon's claim that eating lunch "kind of **feels** like it's short" (Excerpt 4.8). That is, a child can perceive and

quantify duration through how it feels to be in the experience. I noted in Chapter IV that four of my participants described how their experiences felt; however, because I could not articulate this, I chose not to analyze these excerpts. Piaget found that across his participants (ages 4-12) there was an overwhelming agreement that the more engaging the task, the shorter the duration. This seems reflective of Flaherty's (1999, as cited in Evans, 2004) protracted duration and temporal compression—where the amount of information processed can create a varying perception of the duration of an experience. Next, I present some implications from this study for teaching.

Implications for Teaching

Current school-based instruction on time in the United States (detailed in Chapter I) seems to emphasize procedural clock reading and operations with standard temporal units (CCSSI: Measurement and Data, 2020). Kamii and Russell (2012) and Russell (2008) presented specific suggestions on how CCSSI could re-align the standards to better account for children's logico-mathematical and durational reasoning. Here, I add two suggestions for how teachers might utilize the durational understandings their students might have with the temporal content being presented.

The development of durational units that I proposed seems to echo the current progression for other length-based measurement already established by Common Core State Standards (CCSSI: Measurement and Data, 2020). As I described in Chapter I, prior to introducing rulers or other tools of length-based measurement, CCSSI introduces kindergarteners to early measurement concepts—beginning with identifying measurable attributes of tangible objects and moving into comparing and classifying objects based on these attributes (CCSSI: Measurement and Data, 2020). Based on my analysis, I similarly recommend that prior to

school-based instruction in the tools for time measurement (i.e., analog and digital clock reading), teachers begin by providing early elementary children the opportunity to explore non-standard durational measurement, drawing on their existing (re-presented) conceptions of duration as an accumulating of activities completed.

The correspondences between number and duration that I presented indicate a possible correlation between children's development of number and their ability to conceive of duration as a measurable attribute of their experiences. From this, I would advocate for explicit school-based instructional practices that provide children with the opportunity to spontaneously correspond different attributes of their experiences with duration. Long and Kamii (2001) presented activities that could foster the logico-mathematical reasoning necessary for durational measurement. One such activity was allowing children the opportunity to make decisions related to time, for example asking how they might clean up the classroom if they only had a short time to complete it. This question might promote the children's anticipation of the rapidity of their actions, the accumulation of the class's combined activities, their perceived effort or enjoyment, etc., as they quantify the short duration of their potential cleaning experience. I infer that this type of reasoning could promote children's reflection on duration as a measurable attribute of the world (Earnest, 2018a) and support their future executive planning (Earnest, 2018b).

Durational reasoning is not a learning confined to elementary education. As I noted in Chapter I, time often is an independent variable in higher level math and science (Brookes, 2006; Thompson, 2012) and is part of daily life in the United States through scheduling and time management (Earnest, 2018b). The assumptions made by teachers at all levels about their students' conceptions of time and durational reasoning should be noted, as how their students

conceive of time might vary a great deal from their own, adult, conceptions. I now turn to the limitations of this dissertation and future research.

Limitations of this Dissertation and Future Research

The methodology I presented has two primary limitations. First, I theorized time from a Western perspective, while time is inherently cultural. Second, I framed this study in phenomenology, but made analytic decisions to exclude certain data from my final analysis. I discuss each limitation and offer future research for both.

Western Theorization of Time

Children are informally exposed to time concepts from birth. A parent telling their child to "hold on a second" (Evans, 2004) or a family's weekly schedule (Lareau, 2011) are examples of informal time experiences. Such cultural time practices reflect the values and practices of children's families and communities (Lareau, 2011; Levine, 1997). As a White, middle-class adult who was born and raised in the Midwestern United States, my own conceptions of time as an attribute of the world framed my research agenda. Everything from my epistemological review (Chapter I) to my sampling (Chapter III) further situated my own temporal biases.

Recognizing this, an obvious area of future research would be to study other temporal and durational epistemologies and explore a variety of children's conceptions of duration—I am leaving off duration "as an attribute of their world" because of the assumptions it makes about how other children might perceive duration. Broadening the scope of this research, in terms of both the researcher's and the children's background, can benefit the field of mathematics education, as well as educators from around the United States who teach a spectrum of children, not just those with Western perspectives.

Analytic Decisions of Phenomenology

Phenomenological research intends to explore people's experiences through their own descriptions (Creswell, 2013; Hycner, 1999). The open-ended nature of my interviews allowed the participants the opportunity to reflect on the experiences that most resonated with their own durational reasoning. Thus, the variety of different conceptions that emerged was more widespread than what was presented during my analysis (Chapter IV). For example, across his two interviews, 6-year-old Lennon seemed to re-present duration through the three themes that I identified in my findings. Additionally, he considered his and other's ages, his attention, perception, the use of standard durational units by himself and his mom, and physical length as he described the duration of his experiences. I made an analytic choice to present conceptions presented by all of my participants, rather than a deep case study of a single child's durational reasoning. The reason for this decision was to present possible markers (Tzur, 2019) of children's durational conceptions as a foundation for future research. Future research could continue to explore children's conceptions of time with the goal of establishing some conceptual progression or transitions (Tzur, 2019) between these identified markers.

Concluding Remarks

The focus of this dissertation study was on mapping young children's conceptions of duration as an attribute of their lived experiences prior to formal, school-based instruction on time. Utilizing phenomenology (Creswell, 2013; Hycner, 1999) and clinical interview case studies (Clement, 2000), I explored my participants' re-presentations of the duration of past experiences (psychological time; Piaget, 1969) through semi-structured interviews. Through my use of Wolcott's (1994) *Description*, *Analysis*, and *Interpretation*, I identified three common themes across my participants' descriptions: (a) duration as an accumulation of activities

completed; (b) duration as a consideration of a gross quantity; and (c) duration as a result of exertion. From these themes, I conjectured a possible development of durational units (von Glasersfeld, 1981) and correspondence between conception of number (Piaget, 1965) and duration, as well as validating Piaget's (1969) conclusions of children's development of inner duration. Based on the findings, I suggest modifications to how educators might introduce temporal concepts in elementary school and future research on children's durational reasoning.

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