INTERACTING WITH DYNAMIC COMPUTER ACTIVITIES IMPACTS COLLEGE ALGEBRA STUDENTS' MATH ATTITUDES AND PERFORMANCE

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Opportunities matter when it comes to students' math attitudes and performance. Over two semesters, across treatment sections of college algebra students, we implemented a set of dynamic computer activities linking animations and graphs. Across all sections, we administered a fully online survey of students' attitudes toward math. Using mixed methods, we analyzed students' attitudes toward math and their performance on a course final exam. At the end of each semester, we found statistically significant differences between treatment and comparison students' perceived competence toward math. Furthermore, treatment students outperformed comparison students on the course final exam, with statistically significant differences on an item linked to the dynamic computer activities. When students have opportunities to interact with dynamic computer activities, it can impact their math attitudes and course performance.

Keywords: Affect, Emotion, Beliefs, and Attitudes; Post-Secondary Education; Research Methods; Technology

Opportunities matter when it comes to students' math attitudes and performance. Ideally, College Algebra courses would be a place where students could gain opportunities to develop their mathematical competence. However, too often students experience College Algebra courses as gatekeepers rather than opportunity makers (e.g., Gordon, 2008; Herriott & Dunbar, 2009). To address this opportunity problem with College Algebra, we investigate the question: How might College Algebra students' opportunities to interact with dynamic computer activities, linking animations and graphs, impact their math attitudes and course performance?

Strategic uses of educational technology, particularly those that involve opportunities for students to interact with others to discuss mathematical ideas, show promise for promoting math learning for underserved students (Kitchen & Berk, 2016). In an exploratory study, researchers implemented dynamic computer activities in a middle school in a high poverty neighborhood, serving primarily students of color (Schorr & Goldin, 2008). During the intervention, researchers found that students demonstrated both sophisticated mathematical reasoning and positive affect toward math, including emotions such as elation (Schorr & Goldin, 2008). We were inspired by this link between the opportunities for reasoning afforded by educational technology and positive affect toward math, and we aimed to investigate such relationships on a larger scale.

We focus on an aspect of students' mathematical affect—their attitudes toward math (Ding, Pepin, & Jones, 2015; Di Martino & Zan, 2010; Pepin, 2011; Zan, Brown, Evans, & Hannula, 2006). Following Di Martino and Zan (2010), we view students' attitudes as multidimensional, encompassing three interrelated dimensions: emotional disposition toward mathematics, perceived competence toward mathematics, and a vision of what mathematics is (p. 44). For example, students may like math because it makes them think (emotional disposition), may consider themselves as capable of doing math when they practice (perceived competence), and they may view math to be difficult, but fun (vision of mathematics).

Researchers have found emotional disposition and perceived competence to be important dimensions when it comes to university students' persistence in mathematics courses (Bressoud, Carlson, Mesa & Rasmussen, 2013; Ellis, Fosdick, & Rasmussen, 2016). In a large study of university Calculus I students, Bressoud and colleagues (2013) found that students' confidence and enjoyment of math *decreased* by the end of Calculus I, despite the fact that the majority of students completing the survey at the end of the course demonstrated success in the course, as measured by a final grade of A, B, or C. Given the findings from students' experiences in Calculus I, we were particularly interested in courses that served as prerequisites for Calculus I, such as College Algebra.

The study we report is part of a larger research project, for which we designed an intervention providing College Algebra students with opportunities to interact with a set of dynamic computer activities (see Johnson, McClintock, Kalir, & Olson, 2018). We designed the activities to promote students' covariational reasoning, a key form of reasoning that can engender students' conceptions of rate and function (Carlson, Jacobs, Coe, Larsen & Hsu, 2002; Thompson & Carlson, 2017). In the larger research project, we have three aims: to measure students' covariational reasoning (see Johnson, Kalir, Olson, Gardner, Smith, & Wang, 2018), to promote students' positive attitudes toward math, and to promote students' outcomes in College Algebra. Here, we focus on the latter two aims. Using mixed methods, we demonstrate that our intervention resulted in differences in students' attitudes toward math and in students' course outcomes, as measured by a final exam.

Theoretical and Conceptual Underpinnings

Designing Dynamic Computer Activities to Promote Students' Covariational Reasoning

We integrated different theoretical perspectives to design dynamic computer activities to promote students' covariational reasoning. Each of the dynamic computer activities linked computer animations with dynamic graphs. In particular, we integrated Thompson's theory of quantitative reasoning (Thompson, 1994; 2002) with Marton's variation theory (Marton, 2015; Kullberg, Kempe, & Marton, 2017). Drawing on Thompson's theory, we problematized students' conceptions of attributes represented in the tasks (e.g., distance, height). We theorized how students might conceive of task attributes as capable of varying and possible to measure. Drawing on Marton's theory, we problematized students' discernment of different aspects of the tasks (e.g., axes of a Cartesian graph). We theorized what students might discern as we designed differences (e.g., Cartesian graphs with the same attributes represented on different axes) against a background of invariance (e.g., a situation involving a toy car moving along a track.)

Our Perspective on Students' Attitudes Toward Math

With our perspective on students' attitudes toward math, we intend to move beyond McLeod's (1992) categories of beliefs, attitudes, and emotions as comprising distinct components of mathematical affect. We ground our perspective on students' attitudes toward math in the work of scholars aiming to explicate the multidimensionality of students' attitudes toward math (Di Martino & Zan, 2010; Zan et al., 2006). From our perspective, students' math attitudes are not distinct from their emotions. Rather, students' attitudes toward math include students' emotions, as well as other dimensions. Accordingly, we adopt Di Martino and Zan's (2010) multidimensional perspective on students' attitudes toward math. That is, students' attitudes toward math comprise three interrelated dimensions: their emotional disposition toward math, their perceived competence toward math, and their vision of what mathematics is.

Method

The research we report is part of a broader research project taking place across all face to face sections of College Algebra at a public university situated in the downtown area of a large city in the midwestern US. The university serves large proportions of students who identify as students of color and are the first generation of their family to attend college. In fall 2017, 59% of incoming freshmen identified as students of color, and 51% of all freshmen were first generation college students.

Intervention: Dynamic Computer Activities

At the university where we conducted this study, College Algebra is divided in to recitation and lecture components. Each section of College Algebra includes both recitation and lecture components, with a recitation occurring before each lecture. Students always have different recitation and lecture instructors. Typically, recitation instructors are graduate students, and the instructors are faculty members. We implemented a set of dynamic computer activities across selected recitations. In spring 2018, we were still developing some of the activities. Hence, in spring 2018 we implemented five dynamic computer activities across two different recitations. In fall 2018, we implemented seven dynamic computer activities across three different recitations.

To increase students' access to the dynamic computer activities, we developed them on a freely available platform—Desmos—in collaboration with Meyer, the chief academic officer of Desmos. As typical with Desmos, each of our dynamic computer activities involves a series of screens that students move through. There are five main components to each activity. First, students watch a video that depicts a moving object, such as a toy car moving along a track, along with a description of attributes on which the activity will focus (e.g., total distance traveled and distance from a stationary object.) Second, students represent variation in each attribute by moving dynamic segments along vertical and horizontal axes of a Cartesian plane. This design choice was an effort to operationalize Thompson's (2002) discussion of students' use of fingers as tools to represent variation. Third, students sketch a single graph representing a relationship between both attributes. In each of the second and third components, students have opportunities to get computer feedback on their work, a hallmark of Desmos activities. Fourth, students repeat the second and third components for a new Cartesian graph with attributes on different axes. This design choice was inspired in part by tasks developed by Moore and colleagues (e.g., Moore, Silverman, Paoletti, & LaForest, 2014). Fifth, students answer a reflection question which asks them to make sense of another student's reasoning.

We designed the Desmos activities to promote students' covariational reasoning, as well as their conceptions of graphs as representing relationships between attributes that are capable of varying and possible to measure. Through the reflection questions, we aimed to promote students' sense making, rather than rushing to judgments (Johnson, Olson, Gardner, & Smith, 2018). We conjectured that opportunities to engage in covariational reasoning could promote students' productive attitudes toward math and successful outcomes in college algebra.

Treatment and Comparison Groups

Spring 2018 was our first semester implementing the dynamic computer activities. In spring 2018, there were nine sections of College Algebra. One section was a treatment section, and the other eight sections were comparison sections. We selected the treatment section, because the lecture instructor is a co-principal investigator on our larger project, and was willing to implement the Desmos activities before we rolled them out to a larger number of sections.

In fall 2018, there were 13 sections of College Algebra. Prior to the beginning of the semester, we invited recitation instructors to participate in a semester long professional development (PD). The PD had three main aims, to provide recitation instructors opportunities to (1) learn who their students are as humans; (2) to engage in covariational reasoning; and (3) to implement the Desmos activities to promote their students' covariational reasoning. We accepted all recitation instructors who volunteered to participate, resulting in six recitation instructors teaching a total of 10 sections. Hence, in fall 2018, 10 sections were treatment sections and three sections were comparison sections.

Attitude Study

To investigate students' attitudes toward math, we adapted methods used by Pepin and colleagues (Ding et al., 2015; Pepin, 2011). We created an online survey shown in Table 1. The first three questions are the same as those from Pepin's (2011) survey. In addition, we added two questions specific to students' attitudes toward graphs, rather than math in general, because graphs were specific to our intervention.

Table 1: The Attitude Survey

Attitude Survey Questions
I like/dislike math because __________
I can/ cannot do math because __________
Mathematics is __________
I like/dislike graphs because __________
I can/ cannot make sense of graphs because __________

During spring 2018 and fall 2018, we administered the attitude survey four times: Once at the beginning of the semester (in week 2 or 3), and once at the end of the semester (in week 13). To ensure that technology was working and to answer any questions, a research team member was present at each administration of the survey.

Two qualitatively code the attitude survey data, a subset of our team built a coding rubric based on preliminary analysis of pilot data collected in fall 2017. We had initially planned to code students' responses into three categories, the same way as did Ding et al. (2015): Like/Can, Dislike/Cannot, and Neutral/Mixed. However, as we discussed responses from the pilot data set, we determined that we needed two additional categories—Ambiguous and Detached—to capture the scope of students' responses. In our view, these mutually exclusive categories do not position themselves along a line, with positive and negative being at opposite ends of the continuum. Rather, the categories begin and end in a more knotted way, as shown in Figure 1.

Figure 1: Coding for Complexity in Students' Attitudes Toward Math

Table 2 includes sample responses from our data set, with two examples for each of the codes. Responses coded as *Positive* included statements of like/can, while responses coded as *Negative* included statements of dislike/cannot. All of the Positive/Negative responses shown in Table 2 include "because" clauses, in which students explain their response. Not all students included these clauses, which we did not require. We did not code differently when students used a clause to qualify their response. Responses coded as *Mixed* included specific statements of both like/dislike or can/cannot. Often these statements included the word "but" or "however" to indicate the juxtaposition of positive and negative attitudes. Responses coded as *Ambiguous* could cross multiple attitudes. For example, students may state that they like or dislike math because it is challenging. Responses coded as *Detached* separated the mathematics from the humans engaging in mathematical activity. Typically, students responding this way described about math or graphs as things that are "out there," rather than products of their activity.

Table 2: Sample Attitude Survey Responses from our Data Set

Attitude Survey Codes	Example 1	Example 2
Positive: Like/Can	I like math because it challenges me to keep trying and learn more.	I can make sense of graphs because of practice.
Negative: Dislike/Cannot	I dislike mathematics because it is too stressful and complicated.	I cannot make sense of graphs because they don't make as much sense as equations do to me.
Mixed	I only like math when I understand it. When I understand it, I enjoy it. But most of the time I feel like I'm lost. So sometimes I dislike math.	I can make sense of graphs because I know how some functions move by heart. I cannot make sense of graphs because I do not know how all functions move by heart.

Ambiguous	It is challenging.	I look at the points and read info given.
Detached	Math is the universal language.	They (graphs) are just a visual representation of inputs and outputs.

To qualitatively code the attitude survey data, two graduate students served as coders. First, they received training with coding rubric by participating in meetings to discuss the Fall 2017 pilot data set. Next, they independently coded responses, identified disagreements, and then calibrated the disagreements via discussion, consulting with an expert coder if disagreements could not be resolved.

To quantitatively analyze the attitude survey data, we conducted chi square analysis using percent responses in each coding category. In each of spring 2018 and fall 2018, we used the following groups: Treatment (Pre) vs. Comparison (Pre); Treatment (Post) vs. Comparison (Post); Treatment (Pre) vs. Treatment (Post). We examined the data for statistically significant results, then we developed explanations to account for those results.

Outcomes Study: Final Exam

We collected data from students' performance on the common final exam, both in terms of letter grade (ABCDF) and raw score. In addition, we collected item level data for a multiple choice covariation item that we included on the final exam. In the covariation item, students selected a graph to represent a situation involving a relationship between variables. Some of the graphs were unconventional, like the graphs in the dynamic computer activities with which the treatment students interacted. We scored the covariation item as correct/incorrect, with correct responses receiving a score of 1 and incorrect responses receiving a score of 0.

To quantitatively analyze the final exam data, we also conducted chi square analysis using percent responses in each coding category. In each of spring 2018 and fall 2018, we used the following groups: Treatment (ABC letter grade) vs. Comparison (ABC letter grade); Treatment (raw score) vs. Comparison (raw score); Treatment (covariation item) vs. Comparison (covariation item). As we did for the attitude survey data, we examined the final exam data for statistically significant results, then developed explanations to account for those results.

Results

We organize the results into sections devoted to the attitude study and the outcomes study. In each section, we report results by semester: Spring 2018 and Fall 2018.

Attitude Study

At the beginning of both semesters, we found statistically significant differences between students' perceived competence in treatment and control groups, with treatment groups entering with more negative perceptions of their competence. By the end of each semester, each group demonstrated more positive perceptions of their competence, and we no longer found statistically significant difference between treatment and control groups. In spring 2018, we found statistically significant results when analyzing students' perceptions of their competence with graphs. In fall 2018, we found statistically significant results when analyzing students' perceptions of their competence with math writ large.

In the next subsections, we include a number of tables to report results of quantitative data analysis. In tables 3-6, the comparison group is shown in the top row, and the treatment group is shown in the bottom row. In table 7, the treatment group is shown in both rows. In tables 3-7, we report results in terms of percentages of student responses coded in each category. In the narrative, we provide specific numbers for the treatment and comparison groups.

Spring 2018: Attitude. In Spring 2018 the comparison and treatment groups demonstrated statistically significant differences in their perceived competence toward graphs. In the first administration of the attitude survey, the treatment group (n=26) demonstrated a more negative perceived competence toward graphs than did the comparison group (n=112). Table 3 shows the percentages of students' responses coded in each category.

Table 3: Spring 2018 Comparison Pre (top) vs. Treatment Pre (bottom)

	Like/Can	Dislike/Cannot	Mixed	Ambiguous	Detached	X^2, p
I can/ cannot make sense of graphs because ___________	55.4%	15.2%	8.9%	13.4%	7.1%	13.83* p=0.010
	34.6%	38.5%	3.8%	3.8%	19.2%	

By the end of spring 2018, there were no statistically significant differences in students' perceived competence toward graphs between the treatment group (n=19) and the comparison group (n=77). Both groups saw increases in the number of students who demonstrated more positive perceived competencies toward math. Table 4 shows the percentages of students' responses coded in each category.

Table 4: Spring 2018 Comparison Post (top) vs. Treatment Post (bottom)

	Like/Can	Dislike/Cannot	Mixed	Ambiguous	Detached	X^2
I can/ cannot make sense of graphs because ___________	58.4%	10.4%	11.7%	11.7%	7.8%	5.23
	57.9%	26.3%	0.0%	10.5%	5.3%	

Fall 2018: Attitude. In Fall 2018 the comparison and treatment groups demonstrated statistically significant differences in their perceived competence toward math. In the first administration of the attitude survey, the treatment group (n=251) demonstrated a more negative perceived competence toward math than did the comparison group (n=64). Table 5 shows the percentages of students' responses coded in each category.

Table 5: Fall 2018 Comparison Pre (top) vs. Treatment Pre (bottom)

	Like/Can	Dislike/Cannot	Mixed	Ambiguous	Detached	X^2, p
I can/ cannot do mathematics because ___________	57.8%	9.4%	17.2%	10.9%	4.7%	13.86** p=0.008
	54.6%	21.1%	19.9%	2.8%	1.6%	

By the end of fall 2018, there were no statistically significant differences in students' perceived competence toward math between the treatment group (n=204) and the comparison group (n=45). As was the case for spring 2018, both groups saw increases in the number of students who demonstrated more positive perceived competencies toward math. Table 6 shows the percentages of students' responses coded in each category.

Table 6: Fall 2018 Comparison Post (top) vs. Treatment Post (bottom)

	Like/Can	Dislike/Cannot	Mixed	Ambiguous	Detached	X^2
I can/ cannot do mathematics because ________	66.7%	11.1%	11.1%	11.1%	0.0%	5.44
	60.3%	24.0%	10.8%	4.9%	0.0%	

Because of the size of the treatment group in fall 2018, we were able to compare differences between the beginning and end of the semester. Within the treatment group, there were statistically significant differences in perceived competence toward math from the beginning of the semester (n=251) to the end of the semester (n=204). Table 7 shows the percentages of students' responses coded in each category.

Table 7: Fall 2018 Treatment Pre (top) vs. Treatment Post (bottom)

	Like/Can	Dislike/Cannot	Mixed	Ambiguous	Detached	X^2, p
I can/ cannot do mathematics because ________	54.6%	21.1%	19.9%	2.8%	1.6%	11.60* p=0.021
	60.3%	24.0%	10.8%	4.9%	0.0%	

Outcomes Study: Final Exam

In spring and fall 2018, there were no statistically significant differences between the number of students who passed the final exam (ABC) in the treatment or comparison groups. In both spring 2018 and fall 2018, the treatment group outperformed the comparison group. In spring 2018, the final exam passing rate for the treatment group was 70.8% and for the comparison group was 65.0%. In fall 2018, the final exam passing rate for the treatment group was 60.8% and for the comparison group was 59.4%.

The analysis of final exam raw scores revealed similar findings to the final exam passing rates. In both spring 2018 and fall 2018, the treatment group outperformed the comparison group. In spring 2018, the treatment students outperformed the comparison students by 8 points (153.8 vs 145.8). In fall 2018, the treatment students also outperformed the comparison students, but only by 2.2 points (143.8 vs 141.5). The score differences were not statistically significant in either semester.

We found statistically significant differences in students' performance on the final exam covariation item that was linked to the dynamic computer activities. In both spring 2018 and fall 2018, the treatment group outperformed the comparison group. The p value in spring 2018 (p=0.000) was stronger than the p value for fall 2018 (p=0.013).

Table 7: Final Exam Item: Comparison vs. Treatment

Semester	Group	Mean Score	Standard Deviation
Spring 2018	Comparison (n=143)	0.31	0.46

Spring 2018	Treatment (n=25)	0.84***	0.37
Fall 2018	Comparison (n=71)	0.28	0.5
Fall 2018	Treatment (n=258)	0.43*	0.5

Discussion

We provided evidence to support our claim that students' interaction with dynamic computer activities impacted their attitudes toward math and their performance on the course final exam. Our intervention impacted a particular dimension of students' attitudes toward math—their perceived mathematical competence, which has been shown to be an important dimension impacting students' persistence in mathematics courses, such as Calculus I (e.g., Bressoud et al., 2013). We implemented this study in conjunction with an earlier, National Science Foundation funded project promoting students' active learning in College Algebra. Hence, we are encouraged that comparison students also reported more positive perceptions of their competence in graphs and math, respectively.

There are differences in the numbers of students responding to the attitude surveys, with numbers declining from the administration at the beginning of the semester (pre) to the administration at the end of each semester (post). Student attrition was a main cause for the differences. The students responding at the beginning of the semester represent all those students who began College Algebra. The students responding at the end of the semester represent those students who continued to persist in the course. Hence, our pre and post groups are not exactly the same student population.

Students have complex attitudes toward math. Langer-Osuna and Nasir (2016) have called for researchers to develop methods that acknowledge the humanity of students' experiences. By coding students' responses to allow for that complexity—to extend beyond a continuum of positive or negative in students' affect, we have responded to the call. Furthermore, our results suggest that Schorr & Goldin's (2008) findings are applicable on a broader scale, to university students as well as middle grades students. That is, opportunities for reasoning afforded by interactions with educational technology can promote students' positive attitudes toward math.

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References

Bressoud, D. M., Carlson, M. P., Mesa, V., & Rasmussen, C. (2013). The calculus student: insights from the Mathematical Association of America national study. *International Journal of Mathematical Education in Science and Technology*, *44*(5), 685-698.

Carlson, M. P., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, *33*(5), 352-378.

Di Martino, P., & Zan, R. (2010). "Me and maths": Towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teacher Education*, *13*(1), 27–48.

Ding, L., Pepin, B., & Jones, K. (2015). Students' attitudes towards mathematics across lower secondary schools in Shanghai. In B. Pepin & Roesken-Winter (Eds.), *From beliefs to dynamic affect systems in mathematics education* (pp. 157-178). Switzerland: Springer International Publishing.

Gordon, S. P. (2008). What's Wrong with College Algebra? *PRIMUS*, 18(6), 516–541.

Ellis J., Fosdick B. K., Rasmussen C. (2016) Women 1.5 times more likely to leave STEM pipeline after calculus compared to men: Lack of mathematical confidence a potential culprit. *PLoS ONE*, *11*(7): e0157447. doi:10.1371/journal. pone.0157447

Herriott, S. R., & Dunbar, S. R. (2009). Who Takes College Algebra? *PRIMUS*, *19*(1), 74–87.

Johnson, H. L., Kalir, R., Olson, G., Gardner, A., Smith, A., & Wang, X. (2018). Networking theories to design a fully online assessment of students' covariational reasoning. In Hodges, T.E., Roy, G. J., & Tyminski, A. M. (Eds.). *Proceedings of the 40th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1343-1346). Greenville, SC: University of South Carolina & Clemson University.

Johnson, H. L., Olson, G., Gardner, A., & Smith, A. (2018). From soliciting answers to eliciting reasoning: Questioning our questions in digital math tasks. *Colorado Math Teacher*. Retrieved from: https://www.cctmath.org/cmt/2018/8/2/from-soliciting-answers-to-eliciting-reasoning-questioning-our-questions-in-digital-math-tasks-h7lcf

Johnson, H. L., McClintock, E., Kalir, R., & Olson, G. (2018) Networking theories to design dynamic covariation techtivities for college algebra students. In (Eds.) A. Weinberg, C. Rasmussen, J. Rabin, M. Wawro, and S. Brown, *Proceedings of the 21st Annual Conference on Research in Undergraduate Mathematics Education* (pp. 1224-1233), San Diego, CA: RUME.

Kitchen, R., & Berk, S. (2016). Educational technology: An equity challenge to the Common Core. *Journal for Research in Mathematics Education*, *47*(1), 3-16.

Kullberg, A., Kempe, U. R., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM: The International Journal on Mathematics Education*, *49*(4), 559–569.

Langer-Osuna, J. M., & Nasir, N. S. (2016). Rehumanizing the "Other" race, culture, and identity in education research. *Review of Research in Education*, *40*(1), 723-743.

McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: McMillan.

Marton, F. (2015). *Necessary conditions of learning*. New York: Routledge.

Moore, K. C., Silverman, J., Paoletti, T., & LaForest, K. (2014). Breaking conventions to support quantitative reasoning. *Mathematics Teacher Education, 2*(2), 141-157.

Pepin, B. (2011). Pupils' attitudes towards mathematics: a comparative study of Norwegian and English secondary students. *ZDM Mathematics Education*, *43*(4), 535-546.

Schorr, R. Y., & Goldin, G. A. (2008). Students' expression of affect in an inner-city SimCalc classroom. *Educational Studies in Mathematics*, *68*(2), 131–148.

Thompson, P. W. (1994). The development of the concept of speed and its relationship to concepts of rate. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 181-234). Albany, NY: State University of New York Press.

Thompson, P. W. (2002). Didactic objects and didactic models in radical constructivism. In K. Gravemeijer, R. Lehrer, B. van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling, and tool use in mathematics education*. Dordrecht, The Netherlands: Kluwer.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation and functions: Foundational ways of mathematical thinking. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421-456). Reston, VA: National Council of Teachers of Mathematics.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in Mathematics Education: An Introduction. *Educational Studies in Mathematics*, *63*(2), 113–121.