ADAPTING COVARIATION TASKS FOR STUDENTS LEARNING ENGLISH

by

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Adapting covariation tasks for students learning English

Thesis directed by Heather L. Johnson

ABSTRACT

Covariation tasks are used by researchers, designers, and teachers to help engage students into reasoning about functions covariationally. In this thesis I will adapt a covariation task and show how to adapt such tasks for students who are learning English. By adapting a covariation task for students who are learning English I intend to provide more opportunities for students to engage in covariational reasoning.

By adapting a covariation task, I address two related research questions. *How does my teacher's professional perspective shift by adapting a covariation task? How could designers adapt covariational tasks to help students who are learning English?* I share adaptations that I develop, which include using sentence frames to go with the probing questions to engage students in covariational reasoning. Using a constant comparative method (Corbin & Strauss, 2008), I will analyze adaptations that were done. I will also look at the adaptation in detail based on the covariation task and why these adaptions are chosen.

I will attempt to take the journey of a designer and the process behind designing adapted covariation task. With that I will explain the process of keeping a journal and how that simple tool helped shape the design of the adapted covariation task. By adapting covariation tasks, researchers, designers, teachers can potentially have more students engaging in covariational reasoning, including students who are learning English.

Approved: Heather L. Johnson

DEDICATION

I dedicate this to my family. My wonderful partner, Tyler Cross. His support in my writing and dealing with my up and down emotions during this process. To my beautiful daughter who has been supportive of mom always having "to do homework" and missing her gymnastics practice and understanding that I had to do this.

I also dedicate this to the younger me. When I spoke not one word of English and hated writing because of it. But that is how I got my love for math and teaching and how I am now writing this thesis.

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CHAPTER 1

INTRODUCTION

Year after year, students in middle school and high school have difficulties comprehending the concept of functions (Thompson and Carlson, 2017). Functions are key concepts in mathematics, and people often use graphs to represent functions (Leinhardt, Zaslavsky, and Stein, 1990). Ellis (2011) claimed that a key aspect of algebraic reasoning for students is to understand functions and build functional relationships. Building functional relationships in the younger grades can foster students' relational reasoning (Ellis, 2011). Functions are hard to understand sometimes, and building relationships could help students better understand the graphing of a function on a coordinate plane, that goes with functions.

Students learning English are not only trying to learn a language, but they are also trying to understand the relationships within the mathematics curriculum (Kersaint, Thompson, and Petkova, 2013). Because students who are learning English are learning both language and mathematics, they can have challenges with math reasoning. Challenges students may encounter with math reasoning are not because they are incapable of successfully accomplishing tasks. There are systemic challenges that may hinder students' access and opportunity (Kersaint, Thompson, and Petkova, 2013). Furthermore, Moschkovich, (2002) reported students' central aspects of "mathematical reasoning is not only seen as solving procedures and words problems, but also presenting mathematical arguments, participating in mathematical arguments, and mathematical discussions" (p. 192). This assertion from Moschkovich indicates that there are many sides that need to be considered when preparing a mathematics lesson for students who are learning English that include creating opportunities for critical discussion.

Thompson and Carlson (2017) asserted that "covariation is a foundation for functions in mathematics" (p. 421). Even university students have difficulty with the concept of covariation, conceiving of relationships between two quantities that can vary (Carlson, Jacobs, Coe, Larsen, and Hsu, 2002). Carlson (1998) reported that students in a college algebra class, saw evaluating functions "as an item to be substituted" and as a "memorized process"(p. 138). When students think of graphs, they can think of functions as a representation of physical objects as they create graphs (Leinhardt, Zaslavsky, and Stein, 1990).

Students may comprehend quantities in graphs as the two individual entities (Saldanha and Thompson, 1998; Johnson, 2015). Carlson, Jacobs, Coe, Larsen, and Hsu (2002) gave a version of the Shell Centre’s (1985) well-known bottle filling problem to the most high-performing university students (grade of A) in calculus 2. Carlson et al. (2002) found that these students were unable to construct an appropriate graph for the Bottle filling problem. Furthermore providing evidence that many students can reason about the relationship between the height and water but cannot create a graph to represent this.

Johnson (2012) provided high school students an adapted version of the Shell Centre’s bottle problem. Rather than having students sketch a graph, Johnson (2012) asked students to sketch a bottle given a graph and describe how the quantities of volume and height covary, or change together, in relation to the shape of the bottle. The height is changing as the water is filling the bottle making the quantities change together. Considering how important relationships are when talking about covariational reasoning really begs certain questions.

One question is how teachers, researchers, and task designers should approach conversations when talking with students who have only emerging fluency in the English language. The research cited above give some idea of how complex covariation is. To Johnson's (2012) point, research shows that covariation is not easily or readily available but when given the opportunity to engage in covariational reasoning students can in fact engage in covariational reasoning.

Students who are learning English can learn mathematics through conversations with their peers when they are able to participate in the dialogue (Turner, Dominguez, Maldonado, and Empson, 2013). Learners of the English language who are only able to minimally participate in a group discussion can struggle to explain their thinking to others (Turner, Dominguez, Maldonado, and Empson, 2013). For example, when a student comes to the classroom with little to no English and is asked to have a whole class discussion by asking questions, the student does not only need to express the math being taught, but they also need to do so in the English language, which can limit their willingness to participate in the discussion. Mathematical reasoning is not just for students to understand procedures and word problems, but they should also be engaged in mathematical discussion by inputting their own arguments and fully participate. This is important because if students are engaged in discussion they are more willing to learn and not just sit in the classroom and be confused. It is beneficial to the teacher as well to quickly gauge what students are understanding,

Learning mathematics vocabulary is only a part of the story (Moschkovich, 2002). When working with students who are learning English it is important for teachers and researchers to have a sense of what students understand and not be so picky about the particular mathematical vocabulary words that they use (Moschkovich, 2002). For example, when talking about distance, students' use of the correct word, "distance," is not as important as students' understanding what distance means in the given problem. What Moschkovich is claiming is that for students who are learning English to understand math, vocabulary is not the most important concept for them to grasp. More important are the mathematical ideas.

One aspect that can make covariational reasoning tasks challenging for students learning English is the complex language in the tasks. When students are asked to create a graph with the two changing quantities and are able to graph the points correctly that is a great accomplishment. But when students are asked questions to explain their understanding of the relationships they answer with phrases like "this is how it is supposed to look like" instead of explaining the relationship. Like in Johnson's (2012) filling bottle task, students are asked to reason with the idea of how the volume of liquid in a bottle changes as the height of the liquid in the bottle is increased, with the liquid being dispensed into the bottle at a constant rate. Students who are learning English may have challenges using the vocabulary and going straight into questions without first having a sense of the situation. For example, instead of using the term volume, a teacher may ask a question such as "what is happening when soda fills the bottle?" Based on Moschkovich (2002), the vocabulary is not the most important thing, but teachers do have to help students who are learning English to make sense of math questions.

Thompson and Carlson (2017) argue that students need more opportunities to engage in covariational reasoning when it comes to function. I drew from Thompson's and Carlson's (2017) research when framing my research of providing opportunities for students who are learning English to employ covariation perspectives on functions. Like other scholars, I draw on Thompson's and Carlson’s programs of research to communicate what I mean by quantity, quantitative reasoning, variational reasoning, and covariational reasoning. I will provide explicit definitions for these terms in Chapter 2.

It is important for students to have opportunities to work on tasks in which they can engage in covariational reasoning. I use the phrase "Covariation task" to mean tasks in which students have opportunities to engage in covariational reasoning. For example, Johnson’s Ferris wheel task(2016 and 2017) developed a Ferris wheel task that was intended to promote students’ covariational reasoning (Johnson, Hornbein, & Azeem, 2016; Johnson, McClintock, & Hornbein, 2017). Figure 1 shows an interactive Ferris wheel that will be moving through one revolution and will show a relationship between height and distance on a graph. Johnson et al. (2016) claim that the Ferris wheel covariation task is important because students were able to engage in covariational reasoning when working with a Ferris wheel and a series of questions involving height and distance. When discussing height and distance students showed signs of reasoning covariationally (Johnson, Hornbein, & Azeem, 2016; Johnson, McClintock, & Hornbein, 2017). The Ferris wheel task has a lot of terms and a lot of language issues to navigate. It is important because to give language learners opportunities to engage in covariational reasoning, teachers and researchers need to critically examine the language demands of the tasks.

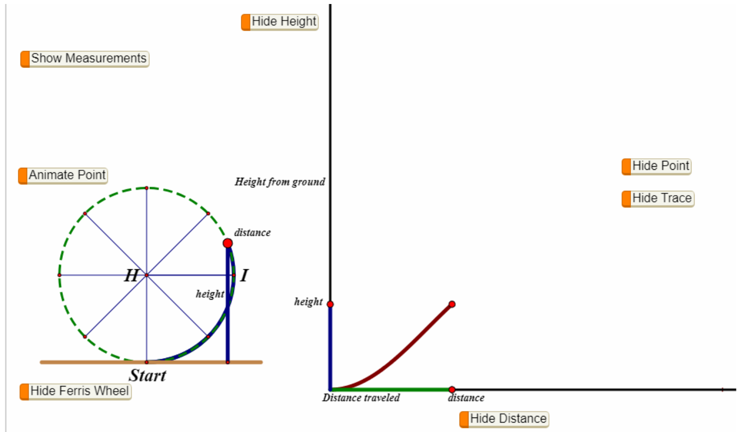

Figure 1: Johnson's (2016) Ferris wheel animation.

In this study, I seek to develop principles to adapt covariation task for students who are learning English. For this research, I will adapt Johnson's Ferris wheel task (Johnson, Hornbein, Azeem, 2016; Johnson, McClintock, and Hornbein, 2017) to provide opportunities for students to engage in covariational reasoning. The purpose of this research will be to aim to answer the following question: *How does my teacher's professional perspective shifts by adapting a covariation task? How could designers adapt covariational tasks to help students who are learning English?*

I organize this thesis as follows. In Chapter 2, I review literature related to covariational reasoning and adaptations in a mathematics classroom. Following that, in Chapter 3 I explain how I adapted the Ferris wheel task for a classroom of students who are learning English as well as how I analyzed my work through a reflective journal. Once I have explained and analyzed how I adapted the Ferris wheel task, in Chapter 4, I explain my perspective related to the methods as well as go into detail on an adapted handout. In the discussion and conclusions section (Chapter 5), I synthesize my thoughts and perspective as related to other scholars who have taken on tasks related to covariational reasoning.

CHAPTER II

LITERATURE REVIEW

The objective of the literature review is to review research related to covariational reasoning as well as to introduce a covariation task and talk about how to include students through communication and language adaptations. First, I begin by defining key terms that will be used in this research. Second, I review research on quantitative reasoning, algebraic reasoning, and covariational reasoning. Third, I review a covariation task. Fourth, I review research investigating how to support language learners' mathematical reasoning and communication. Fifth, I review research on how to support students' who are learning English communication. Lastly, I discuss connecting covariational reasoning and language learners.

Defining Terms

I define key terms in this literature review: quantity, quantitative reasoning, variational reasoning, covariational reasoning, task, covariation tasks, and adaptations.

Quantity. I draw on Thompson's (1994) definition of quantity. Thompson (1994) defines quantity as an individual's conception of some attribute as being possible to measure. With Thompson's definition of quantity, I understand quantity to be something that could be measured based on an individual's conception. A quantity could possibly be given a numerical value. Quantity is someone's conception of the possibility of measuring something, but not necessarily a measurement with a specific numerical value. For example, height is a quantity if a person can think about height as something that is a measurable aspect of some object, and think about a possible way to measure height.

Quantitative Reasoning. I use Thompson's (1994) definition of quantitative reasoning. Thompson (1994) defined quantitative reasoning as being able to operate with quantities and their relationship to one another. Using Thompson's (1994) definition students engaging in quantitative reasoning could conceive distance as something that is possible to measure. For example, a student engaging in quantitative reasoning in Ferris wheel task and using height. The student can reason quantitatively about height because they can reason that height is something that could measure how high or low the Ferris wheel is moving. They could think about numerical values, but they do not have to do so. In the Ferris wheel to help them figure out the measurement of height, they could put numerical values to the height.

Variational Reasoning. By variational reasoning, I mean someone's conception of change in one quantity but not necessarily in a relationship to another quantity. Variational reasoning also brings in conceiving of change in quantities. Thompson and Carlson (2017) define variational reasoning as a person's conception of change in one quantity. For example, when talking about a Ferris wheel the student is looking at distance and height. Students reasoning variationally will be able to conceive of distance changing or height changing. However, students would not necessarily reason about distance and height changing in relation to each other. It would be as if students were reasoning reason about distance and height as two separate entities.

Covariational Reasoning. Carlson et al. (2002) define covariational reasoning to be "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (p.354). For example, when students engage in covariational reasoning they can look at a distance on a Ferris wheel and make sense of distance as changing with respect to the height of the car on the Ferris wheel.

Tasks. Johnson, Coles, and Clarke 2017 define a mathematical task to include "a designer's intended purpose for the task, a teacher's intentions in implementing a task, students' activity in undertaking a task, and artifacts" (p. 2). According to Johnson, Coles, and Clarke (2017) definition of a task, a task can include problem statements, tools, and constructed objects (objects that are designed to support students' reasoning) and include student written materials. The written material comes from the action of teachers and students during the completion of the task. Designers design a task that is not just a problem statement but has a deeper intention for students to engage in deeper learning than producing a correct answer. When students can produce the mathematics given without any real reasoning is an example of having a correct answer where deeper learning looks like students engaged in more than just a procedures but content learning. For example, Johnson, McClintock, and Hornbein (2017) designed a task where the intentions were to engage students in covariational reasoning. The problem statement was twofold: To describe how each quantity (height and distance) was changing, and to sketch a graph relating height and distance. The tool used was the Ferris wheel interactive and the written materials were students' graph sketches and annotations.

Covariation Tasks. Johnson and colleagues (Johnson, Hornbein, Azeem, 2016; Johnson, McClintock, and Hornbein, 2017) report on tasks designed to foster students' covariational reasoning. I refer to these tasks as *covariation tasks.* When students work on covariation tasks, they have opportunities to engage in covariational reasoning. For instance, when working on Ferris wheel task, students could have opportunities to engage in covariational reasoning. For example, when students look at the relationship between the height and the distance of a Ferris wheel they are looking at them as the height increases the distance is increasing and decreasing.

Adaptation. According to Scott, Vitale, and Masten (1998) the word adaptation is used in a classroom by teachers to help students learn the same material and meet the same expectations as other students in the classroom. For example, an adaptation when looking at the relationship between the height and the distance students can be given sentence frames to go with the questions that go with a task without losing the integrity of the content being taught. According to Scott et al. (1998) with adaptations it differs from a modification because a modification is changing of the content to make it easier and sometimes shorter. With a modification, students are given the content but sometimes in shorter ways and involve "diversifying curriculum, individualized instruction, making material modifications, and altering the criteria for grading" are all ways a modification works. (Scott et al., 1998, p. 113). In this thesis I will not be using modifications but instead adaptations.

Early Algebraic Reasoning and Early Function Thinking

Early algebraic reasoning. Students should have opportunities to make connections between quantities and understand what those connections entail (Smith and Thompson, 2008). These connections between quantities entail understanding how the context of the problem can help students make sense of functions. Smith and Thompson (2008) suggested elementary and middle school teachers should change their teaching of mathematics to "prepare children for different views of algebra" (p. 97) through the development of quantitative reasoning. I understand early algebra from Smith and Thompson (2008), as giving elementary students the opportunity to understand quantities at an early age to set them up for success in later math.

Carraher, Schliemann, and Schwartz (2008) explained that students at an early age could start making sense about functions instead of waiting until the secondary years. Based on Carraher et al. (2008), early algebra “done right” can have "students start to think about making mathematical generalizations, however, there is still a lot of work that needs to be done to execute this idea properly in the early years of a student's learning" (p.236). Introducing all students to algebra concepts early on in their life can help to scaffold their thinking about functions. According to Carraher et al. (2008), early algebra done right can look like "building heavily on background contexts of problems" and only "gradually introduce function" notation (p. 236). Carraher et al. (2008) essentially assert that if students are exposed to background context of Algebra problems and gradually introduced to the function notation, then in later years they will be more familiar and comfortable with this type of thinking.

Carraher et al. (2008) offered the Candy box problem as an example of giving elementary students an opportunity to engage in algebra early. The Candy box problem is a problem given to 3rd graders. In the problem, they are given two boxes of candies. Inside the boxes they have the same number of candies. Students are given two problem statements, “box in the left is John’s, and all of John’s candies are in the box, box in the right hand is Mary’s, and Mary’s candies include those in the box as well as three additional candies resting atop the box (Carraher et al., 2008, p. 238).” In the Candy box problem, students are trying to figure out how many candies are in the box. Students think about solving a one variable equation without saying it is algebra. Students are engaged in solving for one variable equation where they are trying to figure out the number of candies in the box because they have an unknown number and they are trying to figure out by using information that is known. As students have the opportunities to engage in reasoning, students begin to make sense of the context of the given problem, and continuing to do that fosters their thinking.

Early functions of reasoning. Ellis (2011) addressed the importance of an early understanding of relationships in functions. Ellis (2011) reported a study in which middle-school students had the opportunity to reason directly with quantities and their relationships to support their thinking about linear and quadratic functions. Ellis (2011) argued the idea was that students could think of functions as building quantitative relationships to "support a covariational perspective and later serve as a foundation to view functions that include a correspondence perspective" (p. 215). Ellis (2011) explained that young children are likely able to think quantitatively in the perspective of relationships.

Students employing covariational perspectives can make sense of functions as quantitative relationships (Carlson et al., 2002). Secondary schools should teach in a way that is mindful of the individual use of student experiences to demonstrate the importance of understanding quantities. This is especially true when teaching in the manner that was previously mentioned, and when working with middle school students this foundation can give them the support they will need in high school to obtain more formal algebraic reasoning. When this type of teaching is done properly it allows for reasoning with quantities and can help support covariational reasoning about functions within young learners.

Algebraic, Quantitative, and Covariational Reasoning

Students' quantitative reasoning can strengthen students' algebraic reasoning. Smith and Thompson (2008) claimed that students typically struggled in their high school algebra class due to the lack of "any linkage between numbers and symbols and the situations, problems, and ideas that they help [students] think about" (p. 96). Smith and Thompson (2008) claimed that elementary and middle school teachers could change their tasks from having students simply operate on numerical values to having students "state general relationships and make inferences from them." (p. 105). If teachers can make this shift in their teaching, then they will potentially notice results in students being able to make sense of the quantitative relationship in functions. Take a covariation task about a Ferris wheel measuring distance and quantity. When the numerical values are taken away and students are asked to make statements about how the two quantities height and distance are changing, students have opportunities to think about functions not as numbers but instead as a relationship between them.

Quantitative reasoning related to functions is important because it helps to explain the way that all students think and process quantitative operations (Ellis, 2007). Ellis (2007) also explained that quantitative operations are just a "conceptual operation by which one conceives a new quantity in relation to one or more already-conceived quantities" (p.440). Ellis' (2007) explanation of conceptual operation helps create understanding for the learning that occurs from a quantitative perspective that was generated based on the prior knowledge of the learner. To give an example of conceptual operation, I use the height and distance of a Ferris wheel. If a student has an understanding of height and then can conceive of it in relation to the distance, that is a conceptual operation. When students can make sense of height in relation to the distance they are engaging in covariational reasoning.

Johnson (2012) claimed that being able to reason with covarying quantities can help to serve as a root for calculus. Johnson (2012) found that a student was able to reason about variation in the intensity of change between covarying quantities through multiple representations. This means that if teachers can teach functions as covarying quantities before Calculus this allows for students to be prepared to have more success in their advanced math classes. Covariational reasoning is not just a stepping stone for Calculus, but instead, something useful for students to understand functions. Overall, Johnson (2012) demonstrated that a student does not need to be in a Calculus class to make sense of variation in quantities involved in the rate of change to reason about functions. In other words, students are able to make sense of two quantities changing together before they reach this level of math.

Both Smith and Thompson (2008) and Johnson (2012) discussed the importance of quantitative reasoning in the development of students' mathematical practices. In order for students to truly learn mathematics, they must have meaning for it rather than just procedures and operations for their thinking. Smith and Thompson (2008) used problems with quantities that did not necessarily have a specific numerical value to have students reason about relationships between quantities. Using problems like Smith and Thompson goes beyond the procedure of marking points to determining the relationship between two numeric quantities. Although Johnson (2012) did use tasks with specific numerical values, her student reasoned about how the quantities are changing without determining numerical amounts.

Another idea that is important to pay attention to is how complex functional thinking is by nature. Ellis (2011) refers to the importance and difficulties of functional thinking as a means to acknowledge that there is deep learning that occurs for students. A functional relationship helps algebraic thinking as the use of a variety of representations in order to make senses of quantitative situations relationally. However, studies have shown that students have a limited view of the function concept. Ellis (2011) claims "students are moving from middle school to high school algebra classes with a faint understanding of functions" (p. 219). Instead through the covariational reasoning approach, Ellis was ultimately supporting students' abilities to express function relationships algebraically, which is important as it demonstrates whether students are learning the mathematics concept while leaving out whether those same students understand the language.

A Ferris Wheel Covariation Task

Next, I will proceed to explain how a covariation task uses relationships between quantities to provide opportunities for students to frame their thinking about functions in a way that demonstrates the depth of knowledge. In the next chapter I will provide a full description of the task. To illustrate, I use Johnson's Ferris wheel task (Johnson, Hornbein, Azeem, 2016; Johnson, McClintock, and Hornbein, 2017).

The Ferris wheel task is a "dynamic Ferris wheel computer task that teachers can use as an instructional tool to help students investigate functions" (Johnson, Horbein, and Azeem, 2016, p. 853). Johnson and colleagues (2016) stated: "the task links an animation of a turning a Ferris wheel to dynamic graphs relating the quantities of height and distance" (p.346).

This is important because in the Ferris wheel task there is language that is being used to describe the relationship between height and distance. Along with the task-specific language, there were other vocabulary words (revolve, change, etc...) that were used requiring students to understand them in order to further their thinking. Teachers and designers need to consider how to make vocabulary heavy tasks accessible to language learners.

Supporting Language Learners' Mathematical Reasoning and Communication

It is important to foster the mathematical reasoning of students who are learning English as an additional language. Moschkovich's (2002) goal was to understand reasons why there is a lack of language-minority students represented in the technical and scientific fields prompted her research. The idea of the lack of diversity begs the question of how to create inclusive learning in mathematics classrooms and just how important this type of learning environment is to the diversity of American society. Moschkovich (2002) used a situated and sociocultural perspective to examine the way in which students who are classified as limited in their English proficiency are learning mathematics by classifying three perspectives of learning: acquiring vocabulary, constructing multiple meanings, and participating in mathematical discourse. Taking these three perspectives into account when designing covariation tasks can help better support the learners who trying to acquire English by creating a culturally inclusive learning environment.

As noted, the three learning perspectives for emerging English language mathematics learners that Moschkovich mentioned are: acquiring vocabulary, constructing multiple meanings, and participating in discourses. Acquiring vocabulary is critical to any learner. According to Moschkovich (2002), an emerging English language learner student faces a different set of obstacles that their teacher needs to be aware of in order to design lessons that our students are able to make sense of. For example, as a teacher, it is important to double-check that the new vocabulary is not being lost in translation. Part of the problem with helping students who are learning English and are beginning to engage in covariational reasoning is that they do not have the language to describe quantities and their relationships.

Moschkovich provided two perspectives related to the way that students acquire a language and mathematics concepts. Moschkovich (2002) discusses her second perspective, which involved students constructing multiple meanings to help cultivate a universal understanding of the concepts. Thompson's (1994) definition of quantity is relevant because it helps all students who are learning English think about quantities as something other than numbers. Students think of these non-numbers as measurable concepts, which is important for students who are learning. It is important for students who are learning English because they can still understand the mathematical concepts without feeling the loss of the meaning of quantity. Students not getting caught up with numbers can be helpful for language learners because they are doing something other than calculating.

Moschkovich's final perspective involves students participating in mathematical discourse practices to further their ability to make sense of new concepts. Creating the opportunity for students to engage in discourse generated opportunities for students to hear multiple perspectives, therefore, allowing them to make multiple meanings in a conversation and then apply this thinking to generate a new form of mathematical communication that makes sense for them. One way to apply Moschkovich's constructs to covariation tasks is to allow students to discuss quantities in a way that explores their relationships rather than just focusing on numerical values. Giving students who are learning English the opportunity to engage in the mathematical discussion may foster their engagement in covariational reasoning. Classroom discussions can provide opportunities for students to engage in covariational reasoning. Students may construct relationships between the two given quantities and talk about the relationship between those quantities.

The importance of discussion for achieving goals set by Moschkovich is similar to Turner and colleagues. Turner, Dominguez, Maldonado, and Empson (2013) articulate the importance of discussion by explaining that it creates the opportunity for students to do things such as "explain and justify solution strategies, pose questions, and articulate connections between mathematical ideas." (p.205). This type of discussion is important to consider because it demonstrates the ideology of discourse and its relevance to assisting students who are learning English in all facets of their education. For example, when students who are learning English can engage in discourse with one another, it may foster more covariational reasoning.

Another point Turner and colleagues make is that students' perception of self will impact the way in which they learn. The researchers indicate that group discussions help to put students in a position to communicate and share ideas (Turner, Dominguez, Maldonado, and Empson, 2013). Understanding how communication and sharing ideas is helpful to students who are learning English should help shape how a teacher sets up instruction for activities for students who are learning the English language, particularly when trying to have students make sense of tasks that are asking them to think using covariational reasoning. Designers and teachers can do this in covariation tasks by making certain adaptations for students to be put in positions to communicate and share their own ideas.

The ideas brought forth by these scholars help to frame additional resources that teachers can use in their classroom when they are attempting to help students make sense of covariational reasoning tasks. Using the three perspectives outlined by Moschkovich can help create an environment that demonstrates the relationship in a manner that all students are able to understand concepts of functions despite their language differences. Creating an inclusive learning environment in terms of the language that is used is not only beneficial to students who are learning English, but to all students alike.

Supporting English Language Learner Communication with Sentence Frames

There are different kinds of support for language learners. According to Donnelly and Roe (2010), certain vocabulary needs to be specified by the designer or teacher if the teacher is the designer of the adapted task (Donnelly and Roe, 2010). However, vocabulary is not the only thing when it comes to adapting tasks, instead, it is something that could be used to support students who are learning English.

Sentence frames are sentences for students who are learning English to help with their discussion. Sentence frames can also be differentiated within student learning (Donnelly and Roe, 2010). Students who are learning English can use them when they are writing or when they are having a discussion as a whole class. Sentence frames create an equal playing field for the student to be able to start the conversation by having them fill- in blanks. For example, a sentence frame that could be used in the Ferris wheel problem could be **Figure 2.**

The ______________ measures _______________. The ________________ measures ___________________.

Distance/ Height **Distance/ Height**

Figure 2: Sentence Frame

Interpreting Donnelly and Roe (2010) a student who needs a lot of support for communicating can be given simple sentences to express the target language. This would look more like sentence frames for the simplicity of the sentences and less fill in the blanks. Figure 2 is an example of less fill in the blanks and more simplicity of sentences. This is helpful for students that have less English vocabulary in their repertoire. These sentence frames help students be a little more direct but still allow for students to feel like they can communicate with one another.

Donnelly and Roe (2010) say that students who are slightly more advanced with their English vocabulary are expected to use more “complex sentence structures, they can receive less fill in the blanks and more of comparing concepts" (p.132). An example using sentence frames in covariation task for students who are slightly more advanced with their English vocabulary would be, students getting sentence frames with not as many helpful simple tenses and words and instead use sentences where they are comparing the two quantities like figure 3. As students learn even more vocabulary they will still be given sentence frames, however, according to Donnelly and Roe (2010) language learners at this level are expected to use the most complex structures, but still would need the sentence frames to formulate ideas to get their communication building.

As the car _______________on the ________________________, the ______________

changes by____.

Figure 3: Sentence Frame for slightly more proficient in the English language students

It is important to note that all students who are learning English are to use the same academic language to communicate. Donnelly and Roe (2010) assert that it is just the complexity of the framing that changes. Sentence frames are for the student to get started on their thoughts of what they would like to express about the problem in a way that allows them to focus more on the mathematics and less on the English language. Based on Moschkovich's work, with covariation tasks, sentence frames can help students who are learning English with their abilities to communicate in a meaningful way as they engage in discourse around relationships with quantities. Students who are learning English as an additional language can also be able to think quantitatively in the perspective of relationships.

Connecting Covariational Reasoning and Language Learners

Researchers investigating students’ covariational reasoning have yet to investigate how students’ language proficiency might impact students’ opportunities to engage in covariational reasoning. This is likely because there has been very little research conducted regarding covariational reasoning and students who are learning English. One perspective that a reflective teacher may take on regarding covariational reasoning and students who are learning English is that covariational reasoning tasks are heavy in the language demands. Covariation tasks are wordy in the sense that they require students to know a lot of mathematical vocabulary. This extensive use of vocabulary generates a lot of questions for these students that they sometimes do not have the English skills communicate.

Many teachers may find it useful to adapt tasks to meet the needs of students who are learning English. For example, in Johnson and colleagues (Johnson, Hornbein, Azeem, 2016; Johnson, McClintock, and Hornbein, 2017) Ferris wheel task, students are asked, "What are the distance and height measuring, “students need to know what distance and height in this problem even means before they can even begin to answer this question. For students who are learning English need to have a classroom shared the meaning of what distance and height are before they can explain the relationship.

Using adaptations like discourse and sentence frames help to bridge most gaps in communication. These adaptations also create opportunities for students to begin to make sense of the mathematics concepts. According to Moschkovich (2002), the purpose of adaptations is to create new support systems for students who are learning English that go beyond the translation of vocabulary and to a place that involves students in communicating about mathematical concepts.

CHAPTER III

RESEARCH METHODS

The objective of this research is to for me to conduct a study on how to adapt a functions task brought forth by an effort to help students who are learning English to better understand mathematical functions in school. For students to understand that functions are relationships between the two quantities and not just a machine where you put an input and the machine shoots out an output, or that the vertical line test tells you if the image of the graph is or not a function. I developed an adapted covariation task and found a viable adaption for English language learners in the ninth grade, so they can move from variation (input, output) about functions, to covariation (relationships between quantities).

To understand the adaptations of the Ferris Wheel Task (2016 and 2017), I will take a qualitative approach to synthesize data and refine conclusions. The primary variables for consideration include English language students of different communication stages, the Ferris wheel task (2016 and 2017), and my notebook where notes, questions, and notes of my reflections were recorded, and my reflections on each adaptation. The above pieces of information are the data that will be analyze later in the chapter.

Questions that will be addressed in the study will be: *How does my teacher's professional perspective shift by adapting a covariation task? How could designers adapt covariational tasks to help students who are learning English?*

Why Ferris Wheel Task?

I used Johnson's Ferris wheel task (Johnson, Hornbein, Azeem, 2016; Johnson, McClintock, and Hornbein, 2017). Johnson and colleagues provide a "dynamic Ferris wheel computer task that teachers can use as an instructional tool to help students investigate functions" (p. 345). The Ferris wheel task encourages teachers and students alike to learn about functions differently than before. For example, in most secondary classes functions are first introduced as input and outputs, every input has an exact output and graph image is a function if it passes the vertical line test. However, students are not really understanding how and why something is a function. Instead, students just know what a function looks like without being able to understand why it works. The Ferris wheel task works with students to look at an animation of a Ferris wheel revolving in one revolution. Students are asked questions throughout to get them thinking about what they see and to see if they start noticing the relationship between height and distance. From there the learning goal would be for students to conceive of a function as a special relationship between quantities.

The Ferris wheel task. The Ferris wheel task, shown in Figure 1, encourages teachers to ask questions like, "Could you predict the height from the base of a car to the ground if you knew the distance the car had traveled within one revolution of the wheel? Could you predict the distance a car had traveled within one revolution of the wheel if you knew the height from the base of the car to the ground?" (Johnson et.al, 2016, p. 345). As students play with the Ferris wheel animation, they have a Ferris wheel turning and have opportunities for relating the quantities of height and distance. As students use the animate point button, the car of the Ferris wheel is represented by a red dot, that red dot moves in a counterclockwise direction. As the Ferris wheel animation is moving counterclockwise there is a graph to the right changing dynamically. Students are asked to sketch what they think the relationship between the variables is and how that graph would look, using only a paper with, x and y-axis and nothing else. Throughout, students need to use vocabulary as well as answer proving question throughout the lesson. Johnson (2014) used these questions for paired interviews with students. I will be adapting this interview protocol for use with a whole group of students.

Table 1: Johnson's (2014) Ferris wheel Interview Protocol (Selected questions)

Johnson's Questions
Introduction Ask them if they remember the Ferris wheel problem from class. Encourage students to talk about what they remember from class.
Identifying Changing Quantities and Describing the Change What changes and what stays the same? What are the distance and height measuring? How is the distance changing? How is the height changing? Does it seem like the height changes more quickly or slowly in some places? Why?
Predicting Relationships between Quantities As distance gets larger, how does height change?
Sketching a graph. Sketch a graph relating height and distance for the Ferris wheel.

Ferris wheel requirements. To participate in the Ferris wheel task, students need to understand the academic vocabulary used in this task. Students need to use different ways to say things with the same connotation. The task also requires students to be able to communicate what their thinking is during the lesson, whether that be, verbally, written form or in a mathematical way (sketching a graph). Students are required to communicate how the variables are moving and why they sketched the graph they choose. A teacher will also need to help students understand the "thing" that is being measured. Teachers should gather evidence of students' conception How do [they] conceive of measuring the height? Where do they see the height? How is this different from the distance? This can help teachers to understand how students conceive of the "thing" that is being measured in the Ferris wheel task. This clarification will help the teacher to better understand where her students are and how to get them where they need to be to conceive of relationships between the two quantities.

Thoughts about the Ferris wheel task for language learners. Students are required to know vocabulary for this task. This task requires that mathematics learners communicate using this vocabulary as well as ask and answer probing questions. This is a beneficial task for all students in creating understanding around the concept of what makes a function and for that reason, it should be accessible universally, especially to those students who are trying to pick up English as their second language. That is the thinking behind adapting this lesson for a classroom that had new English language students could also benefit from a covariational functions task.

English- language development (ELD) lessons must be based on structured language practice that matches students' English Language proficiency levels (Donnelly and Roe, 2010, p.131). I analyzed the Ferris wheel task by looking at the questions and tasks at hand to determine the concepts that students were expected to learn based on the Colorado state standards and the essential comprehension strategies (http://www.corestandards.org/Math/Content/HSF/IF/).

Sources of data

To begin thinking about the adaptations that need to occur for the Ferris wheel task, I started a notebook, where I kept all my thoughts about the task as I read the article, *Investigating Functions with a Ferris Wheel*. I made the decision of keeping a notebook because I wanted to be able to keep my thoughts organized and make sure I did not forget any detail throughout the process of designing a more English language friendly Ferris wheel task.

Journaling was an effective way for me to be able to quickly write ideas down and have all my ideas in the same place. I also prefer the method of writing things down versus typing because I can record my thinking in a less distracting manner. Reflective journaling is a common practice in qualitative research (Ortlipp, 2008). According to Ortlipp (2008) keeping self-reflecting journals is a strategy that can facilitate the research process.

Journal entries included questions to ask me and ask my advisor, Dr. Johnson. Journal entries also consisted of how to proceed with my research and guesses at when certain adaptations should happen. For example, a lesson plan that would be accomplished if I was to try this out in my own classroom and provide for someone else that might also want to use this lesson.

In my notebook, I logged every time I read a new article pertaining to ways of how to adapt as well as when at work when we would have different ELD professional developments. I dated the entries and wrote about the adaptations that would make sense for the task. The entries were as detailed as possible so when I went back and read them I could understand what I was thinking at the time and how to remember what made me think of the adaptation. An entry would include the date, the name of the article, and the ideas that came from that article. Entries were also made up of ideas of what those adaptations would look like in Ferris wheel task. There were thirty entries altogether; some were written really close together some were written a little farther away from each other time-wise. I started January 2017 and I ended February 2018.

To illustrate, I give an example of the very first entry made. The entry was after reading two articles by Johnson and colleagues (Johnson, Hornbein, Azeem, 2016; Johnson, McClintock, and Hornbein, 2017). I wrote an example of how I would make this lesson a lot more accessible for my English language students. I drew the Ferris wheel labeled it with letters and then put a question mark (?). This question mark indicated that I was wondering if I should include points for students to use as a reference for drawing points for when they were asked to draw on their plain graph or would that be too leading for an answer. Below I also listed “height is a function of distance", that was going to be my main objective for students to understand at the end of this. In this first entry, I also included the mathematical language that would be needed for the task and for my students to understand.

Table 2 includes entries that were made and a short description of what was in this entry. Not all entries are included; these entries were chosen because they are the ones that gave me an idea of the different kinds of entries that were made. The following entries were relevant to my research because I was able to understand my perspective as well as also be able understand the adaptations that were going to be used. The entries in the table are the ones that were visited the most when starting the designing process. They are also the first entries made when starting this process as a designer. This to me was the basis for the designing of an adapted mathematical lesson.

The entries started December 27, 2016, during my winter break from school and the semester before I was officially starting my master's thesis class. I started this early to have some time during a break from work to make sure that I had a good understanding of where I wanted to be in January when I met with my university advisor, Professor Dr. Heather Johnson. At first, I made consecutive entries during my winter break of work. But being a full-time teacher, I had to pause this work to complete my daily teacher duties. There was some times in between entries. Entries that had simple ideas, was usually a quick jot down and they were a little closer. Entries on more depth information were usually made during a long weekend or break the year. This process went on until February 7, 2018. There were 10 entries of questions I had for Johnson my advisor. I made 15 entries from reading research and identifying what information I would be using, 5 entries of a sample task with adaptations and scripts.

Table 2. The main Journal Entries

Date	**Entry #**	**Description**
12/27/2016	#1	Objective, vocabulary and how to describe the Ferris Wheel.
01/08/2017	#2	Questions that would be asked of the students. How to phrase the questions.
01/09/2017	#3	List of Sentence frames that were could be given to students to help further their explanations in writing and orally.
01/10/2017	#4	Questions I had for Johnson about the semester coming up and also about the language I would be using.
01/27/2017	#5	A small outline of the lesson that I would be designing and steps of that lesson. This entry took three pages.
07/03/2017	#15	Questions to Johnson
07/03/2017	#16	A list of adaptations being done in great detail.
07/06/2017	#22	A run-through of what the lesson would look like. This being the 4^{th} time that I have re-written it.
07/30/ 2017	#23	Answering the question what data I am going to collect and how.

08/04/2017	#25	A run-through of what the lesson would look like. This being the 5th time that I have re-written it. This time writing it step by step. Of even what I as the teacher would say.
01/13/2018	#27	Questions to Johnson about how to include questions about the original task.
02/07/2018	#30	The shift of my own perspectives as a designer and teacher.

Analysis

In the process of reflecting in the journal entries, I used the constant comparative method (Corbin and Strauss, 2008), I read the whole notebook and then read parts of the notebook that I found to be more information related to what I would like to use. Journal entries were also made when I had thoughts about why these adaptations are important or adapting this task is important. I made a star in the entries that I would need to come back to and further analyze as I am reading the entire book.

I used reflective memos (Corbin and Strauss, 2008) as I read the whole notebook to understand where I was at the beginning of this research. I then read parts to help better understand the adaptations I had come up with and to reflect on the growth of adaptations within the research. While reading I am also reflecting on the entries written and the growth of my own thinking. Using reflective memos, I was able to stay flexible and relaxed so that I could focus on forwarding my thinking (Corbin and Strauss, 2008). Doing reflective memos helps grow the complexity, density, clarity, and accuracy as the research progresses (Corbin and Strauss, 2008, pg. 118). Reflective memos were also known as journal entries.

After reading the entire notebook of entries, I made a star on specific entries where ideas and growth were generated. Journal entries that were starred were because they were important for me to go back and re-read. They usually were journal entries that I knew I was going to use in my adaptations. Examples of a journal entries that were starred were examples of sentence frames that I had come up with and need to be looked at some more. Starred journal entries were usually entries that I wanted to go back and re-read or entries that gave me information for my thesis. The journal entries that I starred indicated questions that I had for myself and for Johnson, indicating moments of introspection for me. I also recorded thoughts with the journal entries that were about articles that needed more clarity. Journal entries that needed more clarity included questions that came up when reading articles about covariation reasoning or sentence frame adaptations. An example of the entries that were recorded would include something along the lines of what I read how I interpreted and how it would be helpful to my design. Other entries that were also important would be earlier ideas of adaptions that I had either forgot about or that I wanted to elaborate more on. For example, when I talked about sentence frames and the example of sentence frames that were going to be used I explained how I modified some sentence frames and then also divided those frames into different stages for the students that I had based on their learning.

I made four reads through passages all together for the data analysis. Read all together, read it all together in the perspective of a designer, read it all together as a perspective of a teacher, then read parts.

How Adaptations Came About From Journal Entries

When reading the articles by Johnson and colleagues (Johnson, Hornbein, Azeem, 2016; Johnson, McClintock, and Hornbein, 2017), I recognized that there were going to be many questions being asked to engage the student in covariational reasoning. Taking this into account when I wrote my first journal entry about the Ferris wheel task I realize I should include something to support student discourse and covariational reasoning. By reading different journal entries I had wrote I was able to make sentence frames that went with the Ferris wheel task. Sentence frames themselves grew from different journal entries I made. Each time I would go in with a different perspective of thinking and that helped to change sentence frames. Altogether only seven journals entries were entries that I put a star on. These journal entries were more important based on the fact of research that would help with the adaptations and the idea of covariation.

CHAPTER IV

RESULTS

How does my teacher's professional perspective shift by adapting a covariation task? How could designers adapt covariational activities to help students who are learning English? The objective of these results is to indicate how my perspective as a designer changed and what adaptations were done. I address my perspective changes and what adaptations were made. I start with a personal narrative about my experiences. Next, I describe the journey of how I came up with some of the adaptations and the ideas that came up with using the original adaptations. I also talk about how certain adaptations were needed and how they would be used as well as the creation of handouts and what my very first handout looked like. To finish I discuss how using my journal entries to grow my adaptations helped me design my ending tasks.

Perspectives Shifts As A Teacher

My perspectives on functions as a teacher before covariation (variation). Teaching ninth grade math, one of the topics that I must teach is functions more specifically how to tell a function on a graph, table, and how to write a function equation. Before starting my master's program, I taught functions using a function machine that we input a number and that machine spits out another number also known as the input and output of the machine. When looking at a graph I would teach students to use the vertical line test to show whether a graph is or is not a function. When students identified a table as being a function they would say because every *input has exactly one output*. That given answer would be enough to say I felt students understood how to identify if a table is a function.

My perspectives shift of functions as a teacher needing to design an adapted covariation task. During my master's program, we talked about what covariation is and how it differs from a function machine approach about functions. With covariation, students are taught how the two quantities being used are related and how they show that relationship. My perspective shift as a teacher was to teach my students to understand relationships and not just "see" what makes a function. I wanted students to shift from thinking about function machines to thinking about relationships. My perspective changed for me to make sure that my students could struggle a little less in Algebra and even in later math courses. My own perspectives on how to teach functions also changed in this process of thinking about my students.

My perspective on why the Ferris wheel task was the task to adapt for English Language learner's students. I was lucky to be presented the Ferris wheel task in one of my courses during my master's from Dr. Johnson herself. I was very intrigued to learn more about the Ferris wheel task and see how to use it with my students. A course after that I was taught the idea of how it was presented to students, we watched a video about two young ladies, who Johnson interviewed. Here is where my interest grew further for covariation and the Ferris wheel task. My thoughts were how I can use this task for my English language students.

The Ferris wheel task is something that most of my students can relate to because they have either been on one or it is something that I can easily explain and show videos about. I like the fact that we ask questions like "Could you tell me the height from the base of a car to the ground if you knew the distance the car had traveled within one revolution of the wheel?" (Johnson et al. 2016, pg. 345) to guide students to understand functions as something of a relationship. I also liked the fact that there was a way to really have students understand see an animation being shown and having a non-numbered graph as well. This being a fun task that every student should be able to experience.

My perspectives on keeping a notebook: what I learned from journaling.

Learning from Analyzing. Going back to and re-reading my journal entries throughout was very helpful. Especially since this developing process was a year process. There was so much learning that was made throughout this year. One of the biggest learning experiences was thinking about the earlier adaptions and the now more concrete adaptions. The research that went into figuring out why I felt passionate about this. I learned how to develop adapted vocabulary, sentence frames, and how to create the handouts for my students who are learning.

Analyzing the journal entries were also, a good refresher from where I had left off. I was still teaching my required curriculum and sometimes life got the best of me. The journal was easy to read and remind myself of the process that I was going through and not be confused about where I had left off. Going back and understanding why I wrote something, what was the thinking that I was going through at the very moment as well learning from the entire process of your own writing and thoughts? I would even say that it also helped me understand why I was even doing this.

Reading journal entries was how I was able to help develop most of the handouts and slides I created for the lesson to be executed. Looking at my earlier entries versus my later entries was also enlightening, as I was able to notice how much I grew. I also enjoyed the entries on different articles I had read so that I would not have to go back and re-read the entire article but instead had an idea of what the article was about and made extra notes on what I learned and where to go back and re-read relevant sections. There is a learning process of writing the entries first, I could write about what I learned from others' writing and bring my ideas to life. Whereas rewriting was also something that helped form my perspective. Nothing comes out perfect that first time you write it and being able to see the product grow from earlier adaptations to what it is now was helpful. Rewriting also affected the adaptations because I was able to understand how some new findings could affect the earlier adaptations. Rewriting was a way to not start from the beginning but to continue and grow each time something important came up. One of the first entries is about including sentence frames for students. The sentence frames were either very vague or gave too much information not creating space for students to think for themselves. When I read the whole article from Donnelly and Roe (2010) I got more ideas of sentence frames. One important idea was we need to first write sentences that express the target language function to develop the sentence frames being used (p. 132). Looking at entries allowed me to gain more knowledge of the adaptions I was using that featured a lot of vocabulary. Also looking at the activities allowed me to reflect on what could be used more for a classroom environment as opposed to a one-on-one teaching environment. I attended a professional development where we talked about how students who are learning English need more practice in the classroom to speak in content-specific vocabulary, which made me consider that students fear to speak in front of others when they are not given the time to practice using the English content, in this case mathematical, language.

Example of one specific journal entry being written and then rewritten. I picked this example because it helped me to understand the idea that went into designing, not just the journal, but the adaptations. The summary is an example of a few things but mainly a run through of what the lesson would be. One reference being how I took notes from Johnson et al. (2016). My notes grew into adaptations that I developed for students who are learning English. I picked this summary of an example journal entry to illustrate my process. This journal entry is personal to me. This is the one entry that is 5+ journal entries as well. Every time I would go back and re-read this entry I would re-write it and continue to grow.

Using one journal entry as an example, I share my process of dissecting an entry and what came from this one entry. I use this journal entry to illustrate where my ideas came from and how I chose the adaptations. This entry helped me decide what I was going to be creating if students after the first day drew a similar picture. I also decided that the picture of what students thought the relationship would be would not be given during the lesson but instead as an exit ticket for the first day. The reason I made this decision was that seeing that journal entry I wanted to have enough time to go over all the students' sketches of their interpretations before moving on to the next part. This was important to me because sometimes you cannot get the opportunity to see all the students' work in a class period so giving yourself an afternoon to do that can help better plan the continuation of the lesson/task.

Summary of the journal entry. Journal Entry (Figure 4) made on February 2nd, 2017, was starred because it was the third time I read Johnson et al. (2016). This time I was reading the article to find any missing pieces of my own learning of investigating functions. I wrote notes down, some were things in bullets and even things that I had already written down but decided to write them down again for the emphasis. There is one bullet "after seeing the animation Johnson asked the student to sketch a graph relating a car's height from the ground. Talk about the importance of." I had a picture of a graph with distance as it relates to height with a note "Show student examples and talks about why are separate and not together?"

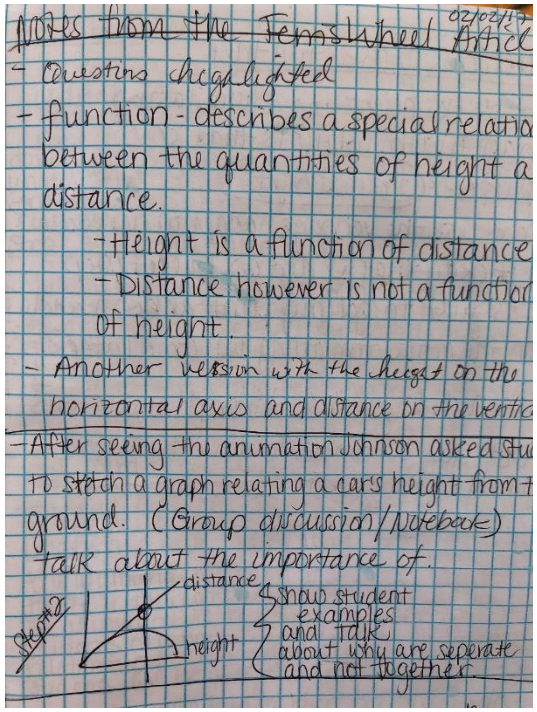

Figure 4: Example of a Journal Summary

Adaptations to the Ferris Wheel Task

Here are the Adaptations that I Went in With. Taking into account students who are new to the country and from different educational backgrounds, I had to assume that some or most students will not even know what a Ferris wheel is in order to be culturally aware. This is a major adaptation to assume they do not know what you will be giving and show in this case what a Ferris wheel is.

Creation of materials themselves. After reading journal entries I starred to create what I was designing, as well as how the design was going to look after what I was reading in each entry. I knew I was going to create handouts from what I was reading in most entries and in those handouts, I would have to think about the vocabulary that was going to be used, the sentence frames for my students to guide their thinking. From re-reading my journal entries I was able to recognize that I needed to include these things as a resource tool for my students who are learning English not just something else that they have to do or busy work. Something more like a collection of all the support tools I had created for them. An example of a journal entry where this growth can be seen is after reading Moschkovich (2002) I interpreted "acquiring vocabulary" as having students be able to express using any words they know but with this to see that students can also be given the main vocabulary words to use and understand while working on the Ferris wheel task.

I did have an approach when designing these handouts. I read journal entries that had to do with vocabulary first and really decided what vocabulary needed to be used and how I needed to use it. After vocabulary was initiated I thought about the questions that needed to be answered and why they need to be answered. Lastly, then came the idea of how these students will use the vocabulary, looking at sentence frames and what those meant in this task. After selecting all of these things I put handouts together and explained how it looked when it was together.

Journal entries and what vocabulary to use. In the first few journal entries, I really tried to understand the vocabulary that was going to be needed for this task. Knowing that there is a lot of vocabulary in this task helped me understand how I needed to organize the vocabulary as well understand what words I needed to use. I want students to not have to get confused with the vocabulary because I used different words that could mean a different thing. Instead in a journal entry, I wrote down all the vocabulary that was going to be needed for this task and chose one word to represent each word needed to complete the task. For example, when we used the word *change*, (when one quantity is changing the other quantity is changing in which way) I really wanted to make sure students could have opportunities to make meaning of the word *change* in this task. After going through all the different vocabulary, I then decided what word to use and a simple definition of words that I knew the students would know.

Now that I have the vocabulary what? Deciding the vocabulary was only one piece of the puzzle. I also had to determine the placement and the ideas of when to introduce each vocabulary word that I was going to be using. I did not want to overload my students with vocabulary words that they did not need in the beginning, but I also needed to figure out where I would be introducing what so that it made sense and it flowed. Right from the beginning, the "main" vocabulary needed to be introduced. I figured out if the vocabulary was going to constantly be used throughout the task it should be introduced on day one. Those vocabulary words would be, height, distance, and revolution. To keep the integrity of the Ferris wheel task, I decided to leave the word revolution because it is a word that Johnson used. I found this to be an opportunity for students who are learning English to be able to learn a new word and be able to use this new word. These three words would be used during the entire task and students needed to be able to understand what that would look like as well on the Ferris wheel. Aside from being able to understand what that would look like they also needed to have a meaning to go with each word. When the vocabulary was presented we as a class came up with shared meaning for each word. I as the teacher was not going to give them the definition of height, distance, and revolution but instead the students were to come up with the definitions.

Questions to go with the task. Just like the vocabulary, there are a lot of guiding questions in this task. For example, this task requires students to think about the Ferris wheel. Again, in another entry I wrote down all the questions that would need to be discussed and discovered that there were even more questions that needed to be asked considering making sure students would understand the point I was trying to get across without obviously giving out answers but more as guiding learning opportunities for me as the teacher to learn what these students have learned. I had recorded in my journal entries ideas of how to present each question. Whether it would be exactly the same or modified to get the point across. In the questions I also wanted to make sure that the cognitive demand of the task was still there. So this helped my thinking about understanding what needed to be asked just like Johnson and what I could instead change to make it clearer for the students.

Sentence frames: to be able to answer the questions. Having questions is important but the way that the students will answer those questions is also important, this is especially true considering that some students' language abilities are stronger than others. The idea of having sentence frames for students to be able to communicate orally and in written form is important. I mean is the purpose of questions is for students to answer, not because they do not know the answer, but because they struggle to produce the language. In the same journal entry as the questions, I answered the questions that I was going to be asking. After doing that I decided what words I would take out so that the students could then insert their own vocabulary. One example was that I would create a blank space for students to insert a vocabulary, word such as height. The specific sentence frames that are part of the student learning experience include active voice to support students' expressing what they see and/ or notice while doing the task. Specifically, students will use these sentence frames to communicate with one another as well as the teacher. Students are also going to have a part in the task to write down their thinking in their journals. The following table shows my adaptations in line with Johnson's (2014) original questions.

Table 3: Johnson's (2014) questions and my adaptions.

Johnson's Questions	Ruiz's Adaptations to Johnson's Questions
Introduction Ask them if they remember the Ferris wheel problem from class. Encourage students to talk about what they remember from class.	My experiences with the Ferris wheel are… ____. Identifying Changing Quantities and describing the change
Identifying Changing Quantities and describing the change What changes and what stays the same?	CHANGE 1. One change is ____ 2. Another change is ____ STAY THE SAME 1. One thing that stayed the same was ______________. 2. Another thing that stayed the same _____________.
What is the distance and height measuring?	Height measures ______, Distance measures____________.
How is the distance changing?	**As the car revolves around the Ferris wheel, how does the distance change?** As the car revolves on the Ferris wheel, the ___________________ changes by _______.
How is the height changing?	**As the car revolves around the Ferris wheel, how does the height change?** As the car on the ___________, the _________ changes by_______________.
Does it seem like the height changes more quickly or slowly in some places? Why?	**If the car revolves faster, does that affect how the distance changes?** **Does the car's speed affect how the height changes?** 1. If the___________ __________er, the d_____________stays the same / changes by____. 2. If the__________ __________________ ________________er, the h_______stays the same / changes by ______________________________.
Predicting Relationships between Quantities As distance gets larger, how does height change?	As ___________________________________, the h________________ ______creases and ____ ______________.

Table 3 cont'd

Sketching a Graph Sketch a graph relating height and distance for the Ferris wheel – what would you choose and why. (probe prerequisite knowledge based on work from class)	Day 1 Exit Ticket: Make a sketch that shows the relationship between the height and distance of the Ferris wheel.

To go into detail about the adaptations seen above. The first part in the introduction the adaptation is an example of an adaptation for a student who is more fluent in the English language. This is so because it only has a one space to fill in the blank. I start by guiding their thinking by saying "My experiences with the Ferris wheel are…." Here the students will finish off the sentences. These students have the vocabulary to finish the sentence.

Notice in the second question where students are asked what changes and what stays the same. Here you see that I did not ask the question and instead actually broke it down into a list form for the students to see the two separate questions. The idea was to chunk the question into two separate questions. What is staying the same and what is changing? Students were given bullet points to get them started if they are more in the intermediate level of fluency. In this part they are getting two adaptations, chunking of the question and a sentence frame starter to engage in the thinking of the question.

The following question on what is the height and the distance measuring I did not put into two different columns because I want students to see the relationship between quantities using in one sentence. Comparing the quantities to each other. Here the fill- in the blanks were height and distance. For students with little to no English this sentence frame works because all they are doing is comparing what is happening with less fill- in the blanks. This is done so students compare content with simpler compare and contrast. You can also see this in the next questions.

The question about the Ferris wheel being slower or faster in some places can be seen as an adaptation of conjugation. Students are going to put in the word slow or fast and not worry about the ending suffix of the word. Instead again students are only comparing the content of the being slower or faster in the Ferris wheel.

One last adaptation seen above in Table 3 is that students as an exit ticket were always given a blank graph where they needed to sketch a graph showing the relationship between distance and height. Use this information to compare their growth throughout the lesson throughout the week.

Putting it together: first handout. In the first handouts, I made a real focus on making intentional note catchers for students' language stages. I wanted to make sure students had space and the opportunity to write, graph, and have something accessible to use as a tool. The first handout was all about the introduction of the Ferris wheel. On the first handout, I had a Ferris wheel vocabulary attached. I wanted students to be able to label the Ferris wheel with the correct vocabulary that we would be using for the task and show on the Ferris wheel the action of revolution, distance, and height. The vocabulary was very intentional for students and words that they are comfortable using. The warm-up was also in this same attachment and the question "What experiences do you have with Ferris Wheels? This warm-up is for me as the teacher to know if students were familiar with what a Ferris wheel is. The students were to see a Ferris Wheel YouTube video https://www.youtube.com/watch?v=E5KCW1TDG-Q . That was a long process to find a video that would be appropriate and would just show a revolving Ferris wheel. I wanted a video were the Ferris wheel was the main attraction and also that it showed how a car could go around one revolution. The one that I found was simple and clearly demonstrated what a Ferris wheel is.

Showing students, a video with a Ferris wheel and talking with students about what a Ferris wheel is could prove essential in case a student has no knowledge of the object. It can also help those acquiring English make sense of a new culture. Also asking a student who has ridden one to show interest in the upcoming task can spark engagement in the content with other students.

Another adaptation that I put questions onto the handout so students could have the opportunity to re-read the question as they were answering, students could better understand their own ideas, also give them the opportunity to read the question instead of just listening to me as the teacher ask the questions. In this same handout was where they could list as many things as they could that would stay the same and what would change in the Ferris wheel task animation. The intention of this is for me to be able to understand what the students are noticing and where I could then take the students. The back side of the same handout there is a picture of the Ferris wheel task wheel where the H is labeled in the middle and the Start red dot is at the bottom.

Before showing the animation, it is important to check for understanding with students to see if they comprehend what they will do. The animation will be playing on repeat, so students can have time to process what the relationship looks like and sketch it on a graph. Students will be doing this individually, so the teacher can walk around and monitor student progress. After five to ten minutes students will then turn to their elbow partner and try to show their sketch and communicate why they chose that as their sketch to represent the animation of their Ferris wheel. Students can choose their best method of communication or a least the one they feel most comfortable with at the moment.

Here the adaption is for students to not have anything in their hands or anything that could distract from watching the animation of the Ferris wheel. Afterward, the teacher will write the direction "Sketch a graph relating a car's height from the ground." Writing this direction on the board will give students the opportunity of reading if they struggle with auditory instructions. Then students will be given time to type into their translators if need be and understand what is being asked. Differentiation of instruction is critical when teaching students who are learning.

Students will be given colored pencils and asked to label on the Ferris wheel picture what the distance and height are measuring. The following questions, "As the car revolves around the Ferris wheel, how does the distance change?", "As the car revolves on the Ferris wheel, how does the height change?", "If the car revolves faster, does that affect how the distance changes?"," Does the car's speed affect how the height changes?", and "As distance gets larger, how does height change?" All of these questions have sentence frames that follow a structure where students can write their response to the questions. For example, one sentence frame that students have available was Figure 3. Students are to fill in the blanks with the vocabulary they feel is appropriate to answer the question given.

Next students can sketch a graph to show a relationship between the height and the distance of the Ferris wheel, with only the x- and y-axis without any labels or numbers.
A whole class discussion can provide opportunities for students to consider many students' sketches. Students will most likely draw sketches of each quantity as two separate graphs. At

this point, the teacher will show a graph of students work to understand why we should not draw them separate and instead of they should be drawn together.

After the class discussion, students will be seated together in groups of three to four. These groups should be comprised of individuals of different cognitive abilities as well as language acquisition. This is to make sure that students are helping one another, and they can communicate with translators and with the help of other levels that can more easily communicate in English. In the groups, they will be asked to "sketch the relationship between the distance and the height." Each student will have their own paper with a one-quadrant graph, but the idea here is to talk to one another and feel comfortable enough to go to another group and be able to share what they found. Once the groups have come to a consensus students will have the opportunity to move and share their findings with another group. Students will be asked to compare their graphs with each other by gallery style presenting to other students from different groups. This will give students the opportunity to be able to talk to one another and if one group is stuck it can help them with generating other ideas. Afterward, another class discussion will come about to bring all the ideas together.

Perspective Change: Now that the Design is Done How Do I Make it Better?

New adaptations. This being a yearlong process I was able to learn from my later entries because I was still learning much about how to adapt a task for students who are learning English. Having some professional development at work I was able to find research on vocabulary word banks. These word banks were to be used with sentence frames, especially for students brand new to this country. I was able to extend the research by being given articles on how to use word banks for each language level of the student.

Those learnings brought me into adding word banks next to the sentence frames to make vocabulary more accessible for students when writing. For example, I went back to the first handout and added the following. "If the car revolves faster, does that affect how the distance changes? Does the car's speed affect how the height changes?"

If the_____ ________ ______er, the _____ stays the same / changes by ___." I added the **"er"** because this was where students would be saying if it was fast**er** or slow**er.** Students can focus on the mathematical vocabulary and not worry about the tense of the word. I also wanted to give students the opportunity to circle because they do not have to worry about writing incomplete sentences but instead can focus on the task at hand. I could have also wrote the word down faster/slower and had the student circle however, when I was reflecting on journal entries I felt like that would be to guiding and again with adaptations I do not want students to lose the integrity of the lesson but instead just have some simple assists to answer questions.

Connections to My Research Question

I used the process of journaling as a resource tool to learn from and grow from. With each journal entry, I made a deeper connection to my previous entries. The perspective of me as a designer shifted my thinking of how adapting covariation activities was a must. The need for students to have this concept of functions not just as a "thing" that goes in and out of something. But instead "something" as a relationship that can be seen by understanding how quantities relate to one another. As a designer going through a process of learning how to adapt covariation tasks helped me to use that learning to adapt even more mathematical tasks for students who are learning English. I am not a teacher who received her schooling in English Language immersion, but I saw a need for the population of students that are in my classroom. I wanted to make the task accessible for every student

CHAPTER V

DISCUSSION

Being the designer was a great learning experience! I was able to understand how to make recommendations as a designer and as a teacher as well. I was also able to see the benefits of reflecting on the work that I was completing. Understanding what changes would come about and why those changes needed to happen.

Language Adaptations for Covariation Tasks

Many have researched covariation but researchers have yet to address language adaptations for covariation tasks. More and more students in the US are learning English as an additional language, and we as researchers, designers, and teachers need to take this into account. My research contributes to covariational reasoning and task adaptations because I bring this together. Johnson's Ferris wheel task (Johnson, Hornbein, Azeem, 2016; Johnson, McClintock, and Hornbein, 2017) helped engage students in covariational reasoning. I contribute to this research because I take into account the intersection between the cognitive and language demands of this task. By using the adaptation of sentence frames, I am taking the interview questions and applying sentence frames that go with these questions. By doing this the cognitive demand of engaging in covariational reasoning remains. Students who are learning are given the opportunity to also engage in covariational reasoning with help on the language. The demand of engaging students in covariational reasoning can still be achieved with sentence frames. The same questions were asked and the order in which Johnson et al. (2016 & 2017) presented the Ferris wheel task was the same as well.

Drawing on Moschkovich (2002) I was able to look at the ideas of the Ferris wheel task and adapt them for multilingual students learning English. From Moschkovich (2002), I took the idea of not focusing on only the vocabulary but have students understand what height and distance meant in the Ferris wheel task. I drew on Turner et al. (2013) to help frame the way in which I was going to take students to communicate in a whole group discussion and engage in discourse. Turner et al. (2013) also helped shape the way I proved questions by thinking about the students whom I was going to teach. Moschkovich and other researchers have implemented adaptations in mathematics classrooms to support students who are learning English. However, researchers have yet to show how these adaptations could be used in a classroom to foster students' covariational reasoning. From Donnelly and Roe (2010) I was able to construct sentence frames for my multilingual students who are learning English. Donnelly and Roe (2010) have done adaptation of sentence frames but not talked about how to use them in a math classroom.

Drawing on Moschkovich (2002) and Turner et al. (2013) helped me to adapt Johnson's Ferris wheel covariation task, to add to covariation literature. By adapting the vocabulary and the probing questions, I intended to provide more opportunities for students who are learning English to engage in covariational reasoning. The work that I have done in adapting the Ferris wheel task extends to other covariation task because the adaptations of vocabulary can be used with any quantities. Also sentence frames can be given to students to use in any covariation task. Researchers studying covariation can focus on opportunities for students learning English by thinking about these simple adaptations. If when they are designing covariation tasks they can keep these adaptations in mind, more students could engage in covariational reasoning.

Personal Reflections

I looked at covariation and adapted a task to fit better for students who are learning English as a second language. This was important to make happen because again we have so many different languages and students who are moving to the United States with little to no idea of the English language. They are moving here as older students, and I cannot imagine how challenging it would be to both have to learn a new language and understand mathematics. When I started to learn English, I was young. I remember getting confused on a few words. For example, when I would hear the word plane I would imagine an airplane and it took some time for me to understand that my teachers meant the Cartesian plane.

Adapting covariation tasks is helpful for students who are learning English because students can really focus on a few key vocabulary words and really dive into what those words mean, while still demonstrating their understanding of the content. Many of these activities require a lot of questions to get the students thinking and talking. If a student who barely speaks English wants to talk or understand what is being said, teachers and researchers must come at it in a different way. Making covariation accessible can provide many more students with opportunities about quantities, and in turn develop math thinking useful for gatekeeping math courses such as algebra or calculus.

Recommendations for Designers and Teachers

When trying to design a curriculum I recommend being very patient, equally thorough, and believing in the work. The biggest helper I had in this process was my journal. I highly recommend keeping a journal of your work for the reasons previously mentioned. For me, the journal helped keep my thoughts and learning all in one place and semi-organized. Keeping a journal was helpful because it is something that I was able to carry around with me and have easy access to. For example, teachers have many professional development sessions, often around multilingual learners. Because I use a different laptop at home than at work, it was easier to take my journal along with me for all the professional developments around our multilingual students learning English. I also enjoy writing over typing.

The journaling process was something that helped with how to grow from my own learning as a designer. After new knowledge from readings or professional development helped to better make adaptations for multilingual students learning English. As a teacher journaling helped with the process to grow in my own classroom with other mathematical concepts I was teaching. For the same process of re-reading the research, I was able to further my understanding when it came to certain material that I may have understood differently or missed the first time.

Journal Entries

Journaling was an easy access to write about anything that pertained to my design tasks. Reading many articles was something where I had to get information that I would be using. I was able to read an article and write a summary of the article as well as makes notes of information that I would need to look at again to further study. Sometimes highlighting is not enough the journal entry that I would make about a certain article would take the highlighted part and further dissecting what I interpreted from that as well how I could use it to further my designed task.

Questions that I needed to get answered pertained to the best way to use the journal. Again, having these all in the same place would help get questions answered in a faster manner instead of trying to remember everything and maybe forget things along the way. Looking at the questions was something else that was very interesting to go back and read and reflect on the learning that I had made for myself as I was able to answer a lot of my original inquiries.

Journaling was also something that I needed to happen while I was having this learning processes for a place to put my ideas into as well. The ideas on how to begin the task and the whole entire process of what the task entail and the adaptations that I was going to implement. The process of journaling was helpful but also the simple fact of reading something more than once from a different perspective allowed me to understand it better. Having the journal to reference was extremely beneficial. I could go back and read something in the perspective of a student wanting to gain knowledge, a teacher wanting to learn to teach this to a group of students, and as a designer trying to design an accommodated covariation task.

Reflections as a Designer

Changes Made. It is important to reflect on your own learning as a designer. Going back and reading was also very beneficial because I was able to see my own learning experience throughout. The main one that I learned from was at the school I teach, we have always been told we should incorporate sentence frames for students. Sentence frames are almost second nature in my teaching. However, on November 21st, 2017 we had a professional development with all the teachers and this teacher was able to enlighten me on the use of sentences frames. She was able to point me to an article that she was reading in her master's program that explained sentence frames. I was able to change my teaching that students should not have a one size fits all sentence frames, but instead also have sentence frames for each level of language for each English language learner. With those sentence frames, we should also have a vocabulary word bank for students to go back and use as a resource. That was one of the changes that were made by going back and reflecting.

Going back and reading my notes I realize that I should be more intentional on how to formatively assess the students. Being that students who are learning English may or may not have all the vocabulary that they need to understand the material. It is more important to understand more content every day as that is a real testament to the way I teach. One of the reflected changes I have is that instead of having students just sketch one time what the graph of the Ferris wheel I would every day give them an exit ticket that was I as the teacher could quickly learn what their interpretation of the material was for that day. I also thought about how I should extend this task for a period of days and not rush through the lesson.

Like exit tickets, I looked at how I want to learn what the students learned. Originally, I was thinking of a formal quiz and grading it that way, but I changed my mind. Going back and reading and reflecting I decided that instead we should do more one sketch and have a set of sentence frames that each student would have to choose on describing the relationship between distance and height. Also explaining why that would make sense for the above relationship using the Ferris wheel to prove their sketch.

Adapting for Students Who Are Learning English

Teachers if you want to adapt tasks or lesson for your students who are learning English I strongly suggest that you remember a few things. First and for most adapting task or lessons is to help students understand and produce the language. Provide students with the opportunity of being able to express themselves even if they do not have the vocabulary to do so. Moschkovich (2002) and Turner et al. (2013) have said that having students who are learning English should engage in mathematical discourse. Even though Moschkovich asserts that vocabulary does not have to be everything, Moschkovich (2002) and Donnelly et al.

(2010) both agree that the classroom should have a shared meaning of the words they will be using.

To connect to covariational reasoning, students should have a shared meaning for the quantities represented in covariation tasks. I drew on Thompson's (1994) definition of quantity when adapting covariation tasks. To be specific, quantities are something more than numbers. They are things that are possible to measure. When implementing Johnson's Ferris wheel task in a whole class setting, teachers and students should come up with a shared meaning for what height and distance could possibly measure.

Do not think that it must be a long process like I had mine be. However, I would start small and grow from that. Meaning I enjoyed doing this for one task that I thought would very beneficial for my students. I also created the space for me to implement and tweak my activities to be more inclusive. From there I became confident in doing this again with another task as well as understanding the process of writing, reflecting and changing. Teachers can use what I have done as a designer to use some of the same adaptations just change the adaptations to something that is related to the task you are looking at doing.

One other adaptation I would have included was the TPR (Total Physical Response). Students when learning the vocabulary would get up and act out the vocabulary or what distance looks like and height. For example, on the first day of introduction student would wait for the teacher to say distance, the students would repeat distance and then they would stand, and we would act out what distance is.

Conclusions

To conclude, these are the three things a designer and teacher should keep in mind: adaptations for covariational activities, maintaining a journal, and reflecting on your work. Adaptations for covariation activities are necessary so all students can be exposed to this type of thinking and learning. Adapting covariational activities is not easy, but by keeping the perspective of students who are learning English in mind, this kind of task proves to be possible. Keeping a journal can be beneficial for reasons beyond adapting lessons. Journaling provides quick access to your own individual learning later down the road. Journals also serve as a tool to help when reflecting on your work. Journaling helps to illuminate the product and reflect on how to make it better for the next time. This adventure of adapting the Ferris wheel task was very eye-opening around how to make math accessible to not only students who are learning English, but to every student in a math classroom.

Having a set of tools that support students who are learning English with acquiring covariational reasoning could be beneficial for all math teachers. It could allow for more students to understand the relationships with quantities instead of just the procedures and operations to solve mathematical problems. With discourse and adaptations, teachers/designers can go beyond vocabulary and give tools to help students who are learning English engage in covariation task.

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