NETWORKING THEORIES TO DESIGN A FULLY ONLINE ASSESSMENT OF STUDENTS' COVARIATIONAL REASONING

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Networking theories of different grain sizes, we designed a fully online assessment of students' covariational reasoning. With this assessment, we intend to produce a viable means of measuring students' mathematical reasoning using methods other than clinical, task-based interviews. The assessment is fully online, and readily accessible across different types of devices. We outline design aspects across and within the assessment items and provide three theoretically based design principles underlying the design of the assessment. Through this research, we contribute to the development of new theoretical approaches to investigate and assess complexities of students' mathematical reasoning.

Keywords: Assessment and Evaluation, Research Methods, Reasoning and Proof, Technology

We address the problem: How can a fully online assessment be developed, in place of clinical interviews, to make inferences into students' covariational reasoning? Building from the work of Norton, Tzur, and colleagues (Hodkowski, Hornbein, Gardner, Johnson, Jorgensen, & Tzur, 2016; Johnson, Tzur, Hodkowski, Jorgensen, Wei, Wang, & Davis, in press; Norton & Wilkins, 2009), we designed an assessment to measure undergraduate college algebra students' covariational reasoning. We extend existing research in two ways. First, we developed a fully online assessment, rather than a paper and pencil tool. Second, we designed our assessment to measure covariational reasoning, rather than multiplicative or fractional reasoning.

We aimed to not only assess students' covariational reasoning, but also to distinguish *gradations* in students' covariational reasoning (see also Johnson et al., in press). Building from the work of Johnson and colleagues (Johnson, McClintock, Hornbein, Gardner, & Greiser, 2017), we networked, or interweaved, theories to design assessment items. Interweaving Thompson's theory of quantitative reasoning (Thompson, 1994; Thompson & Carlson, 2017) and Marton's variation theory (Kullberg, Kempe, & Marton, 2017; Marton, 2015), we designed within and across assessment items. To distinguish gradations in students' covariational reasoning, we drew on Tzur's method of fine grain assessment (Tzur, 2007). Using the fine grain assessment method, we designed our assessment with the intent to investigate how students' opportunities to conceive of variation in individual attributes might foster their covariational reasoning. We outline design aspects within and across assessment items and provide three theoretically based design principles underlying the design of the assessment.

Theoretical and Conceptual Framework

Networking Theories to Design Within and Across Assessment Items

To design within and across assessment items, we networked Thompson's theory of quantitative reasoning (e.g., Thompson, 1994; Thompson & Carlson, 2017) and Marton's

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variation theory (e.g., Kullberg et al., 2017; Marton, 2015). Marton and colleagues (Kullberg et al., 2017; Marton, 2015) identify two key aspects of variation theory: Discernment and variation. For students to discern critical aspects of an object of learning, they need to experience variation (difference). Specifically, learners should experience variation in critical aspects across a background of invariance. Then learners should repeat experiences across different backgrounds. In our set of assessment items, we designed for variation (difference) within assessment items (a background of invariance), then across assessment items (different backgrounds).

To explain the object of learning—covariational reasoning—we appeal to Thompson's theory of quantitative reasoning (e.g., Thompson, 1994; Thompson & Carlson, 2017). In this theory, Thompson draws on students' conceptions of attributes to explain students' mathematical reasoning. In particular, some attribute is a quantity if an individual conceives of that attribute as possible to measure. By covariational reasoning, we mean students' conceptions of relationships between attributes that are capable of varying and possible to measure. For example, consider a toy car moving around a square track. A student engaging in covariational reasoning could conceive of a relationship between the toy car's total distance traveled and the toy car's distance from a center point on the track.

Across our assessment items, we included situations incorporating attributes having different kinds of variation (change). To clarify, we distinguish this use of variation (change) from Marton's use of variation (difference). Across the items, we varied not only the direction of change in attributes (e.g., increases or decreases); we also incorporated variation in unidirectional change in attributes (e.g., "increasing" increases or "decreasing" decreases).

The Fine Grain Assessment Method

To distinguish gradations in students' covariational reasoning, we adapted Tzur's (2007) method of fine grain assessment. When using fine grain assessment methods, designers begin with items that include no supports, then move to subsequent items including increasing amounts of supports. Because items including no supports appear before items with supports, assessment designers have the potential to investigate different levels, or gradations of students' reasoning (see also Hodkowski et al., 2016; Johnson et al., in press). In particular, we intend to use this assessment to investigate how opportunities to conceive of variation in individual attributes might foster students' covariational reasoning.

The Covariational Reasoning Assessment

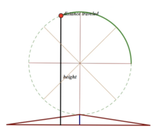
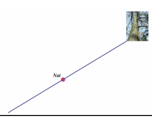
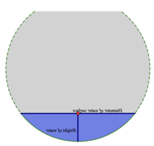
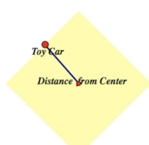
Table 1 provides a map of the covariation items. The covariational reasoning assessment contains four assessment items. Each assessment item contains four question groups.

Assessment Items and Question Groups

Assessment items. Each assessment item incorporates a situation involving changing attributes. We incorporated variation (difference) across the collection of assessment items. The collection incorporates different kinds of attributes (e.g., height from the ground, total distance traveled, diameter of the water surface of a fishbowl, and distance of a car from a center point). Furthermore, the collection incorporates different kinds of variation (change): Variation in the direction of change and variation in unidirectional change (e.g., an "increasing" increase).

Question groups. Each assessment item contains four question groups. Question group 1 serves as a member check, to assess if students comprehend the situation. Question group 2 serves as the first assessment of covariational reasoning: Reasoning without support. Question group 3 provides support for students' covariational reasoning. Question group 4 serves as a second assessment of covariational reasoning: Reasoning with support. We designed our question groups based on Tzur's (2007) method of fine grain assessment. If students respond correctly to Question group 2, they will move to a new assessment item. If students respond incorrectly to Question group 2, they will move to Question groups 3 and 4.

Table 1: Map of the covariation items in the covariational reasoning assessment

Assessment Item	Question Group 1: Member Check	Question Group 2: Reasoning without support	Question Group 3: Supports for Reasoning	Question Group 4: Reasoning with Support
Ferris Wheel  Nat, Path, Tree  Fish bowl  Toy Car 	Play Video Statement of Attributes Do you understand the situation? •Yes •No •If No, explain what makes this situation difficult to understand •If Yes, move to Q2	Play Video Select a graph that represents a relationship between the attributes •A •B •C •D Explain why you chose the graph you did •If Incorrect, move to Q3 •If Correct, move to next assessment item	Video of dynamic points moving along axes Describe how attribute 1 is changing Video of dynamic points moving along axes Describe how attribute 2 is changing	Play Video Select a graph that represents a relationship between the attributes •A •B •C •D Explain why you chose the graph you did

Design Principles

Assess for a Spectrum, Rather than a Switch

We use the analogy of a spectrum versus a switch to communicate our work to move beyond binaries in assessing students' covariational reasoning. Applying Tzur's (2007) method of fine grain assessment, we designed to distinguish gradations in students' covariational reasoning. We anticipate our gradations to be compatible with, yet perhaps not identical to, levels of covariational reasoning put forward in the framework of Thompson and Carlson (2017).

"Practically" Apply Theories to Design Assessment Items

We aim to interweave and apply theories to do practical work of assessment design. With Thompson's theory of quantitative reasoning, we designed assessment items in which students could have opportunities to conceive of attributes as possible to measure and capable of varying. Therefore, we interrogated the types of attributes and variation (change) included in the assessment. With Marton's variation theory, we designed for variation (difference) within and across assessment items. Within assessment items, we incorporated different types of graphs (e.g., linear/nonlinear). Across assessment items, we incorporated different backgrounds (e.g., a Ferris wheel, a toy car, etc.) and different types of variation (change) in attributes (e.g., variation in direction of change and variation in unidirectional change).

Leverage Technology to Promote Access and Opportunity

We leverage multiple technological affordances to promote greater student access and opportunity. Students can access the covariation assessment via a computer, a tablet, or a mobile phone. We created the animations following design factors for effective educational multimedia, including multiple representation types, pacing, cueing, and user manipulation (Plass, Homer, & Hayward, 2009). The assessment incorporates various multimedia, such as original video animations, which provided multimodal representations of dynamic graph attributes.

Concluding Remarks

Networking theories, we interweave variation (change) and variation (difference) in assessment design. We leverage technology to apply Tzur's method of fine grain assessment. Overall, we are working "practically" to network and apply theories to design accessible tools to investigate and assess gradations in undergraduate students' covariational reasoning.

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