THE USE OF QUESTIONS AND GESTURES IN DISCOURSE DURING PROBLEM-BASED INTERVIEWS

by

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The Use of Questions and Gestures in Discourse During Problem-based Interviews

Thesis directed by Professor Heather Johnson

ABSTRACT

This research thesis examined the possible links between teacher questions and how students responded during seven problem-based interviews each consisting of a researcher and two 7th grade students. The interviews were conducted by another researcher at a mid-western middle school. This study focused on the types of questions the researcher asked (during the interviews), the resources (verbal and non-verbal) students used to answer the questions, and the manner in which students interacted with the researcher and their partner. Video recordings and transcripts of the interviews were coded for questions and gestures, revealing five types of questions, and five types of gestures. This study found that the type of questions asked of students during the interviews guided their thinking and explanation of the problem they were presented. Students used gestures to support their answers.

The form and content of this abstract are approved. I recommend its publication.

Approved: Heather Johnson

DEDICATION

Historically, the United States has been a melting pot where people come seeking refuge, solace and freedom. Many of them come with very little knowledge of the English language and that puts their children at a disadvantage in school. Additionally, there are children growing up in the United States that are disadvantaged because they grow up in low socio-economic communities. This work is dedicated to all the teachers who have raised the bar in mathematics education by pushing their students to a higher level of understanding through discourse in their classrooms.

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CHAPTER I

INTRODUCTION

According to the Programme for International Student Assessment (PISA) 2012 report, the United States, while spending more per student than most countries, ranked 27th in mathematics among the 34 participating countries. Reportedly, students from the United States are weak in applying mathematical knowledge when solving real-world problems (PISA, 2013). Furthermore, the student population in the United States is becoming more diverse, in particular the percentage of the Latino/a and Asian/Pacific Islander student population is increasing (Barbu & Beal, 2010; National Center for Education Statistics). Given the growing diversity of students, educators and policy makers are faced with the dilemma of meeting the needs of an increasing number of diverse learners (Walshaw & Anthony, 2008) while enhancing the cognitive knowledge of students.

Teaching that is focused on mathematical concepts and allows students to wrestle with the mathematics improves student achievement (Moschkovich, 2012). Moschkovich (2002) noted that learning mathematics is more than just "developing competence in completing procedures, solving word problems, and using mathematical reasoning" (p. 192); she further stated that it involves learning sociomathematical norms, such as how to present mathematical arguments, and participate in classroom discussions (Yackel & Cobb, 1996).

Through classroom discourse, students can develop a "mathematical disposition" (Walshaw & Anthony, 2008, p. 520), learn ways of thinking about, reflecting on, and clarifying mathematical ideas. The more students express their ideas, the more visible their mathematical reasoning becomes and the easier it is for the teacher to understand what they know “and what they need to learn” (Walshaw & Anthony, 2008, p. 526). Walshaw and Anthony (2008) suggested that discourse “centered on powerful ideas” (p. 517) can make a difference in students’ learning. However; such discourse, while effective, is often challenging to obtain. In particular, educators want students to learn the content, while allowing the students’ ideas to guide the discussion (Sherin, 2002). In other words, teachers must strive to support the process of the classroom discourse while guiding the students toward an understanding of the content.

Reform based teaching, includes discourse that caters to the needs of more students, may hold the key to achieving more equitable results in our mathematics education (Boaler & Staples, 2008; Sherin, 2002; Walshaw & Anthony, 2008). However, the transition from a traditional teacher-directed classroom instruction to a reform type student-centered classroom does not come easy and is demanding on teachers (Sherin, 2002). Teachers must be able to incorporate questions, in their instruction, that guide student learning of mathematical content, set expectations for sociomathematical norms, and help students develop conceptual knowledge (Boaler & Brodie, 2004; Franke, et al., 2009; Hiebert & Wearne, 1993; Purdum-Cassidy, Nesmith, Meyer, & Cooper, 2014; Sherin, 2002).When participating in mathematical discourse, students use a variety of resources; such as gestures, mathematical representations, objects, everyday experiences, and language; to help them communicate their understanding of mathematical concepts.

Meeting the needs of a diverse student population is a challenge faced by teachers of mathematics. For this study I examined students’ communication with each other and the researcher during problem-based interviews. In particular, I asked the following research questions: 1) What type of questions did the researcher ask and how might the type of question afford or constrain how students explained their thinking? 2) What resources (such as gestures) or funds of knowledge (Moll, Amanti, Neff, & Gonzalez, 1992) did students use to explain their thinking and how did they use those resources to communicate their understanding of the mathematics, support their thinking, and justify their responses? 3) In what patterns of communication did the researcher and students engage?

CHAPTER II

LITERATURE REVIEW

Discourse and Reform in Mathematics Education

Reform in mathematics education raises the expectation for discourse in the classroom (Hiebert & Wearne, 1993). Gee (1991), defined discourse as “more than just language” (p. 142), it is the “socially accepted…ways of using language, of thinking, feeling, believing, valuing, and of acting that can be used to identify oneself as a member of a socially meaningful group” (p. 143). Furthermore, discourse involves the use of “language and other symbols systems to talk, think, and participate in the practices that lead to…learning” (Moschkovich, 2007, p. 28). Moschkovich (2002) emphasized that “gestures, artifacts, practices, beliefs, values and communities” (p. 199) play an important role in the way students, in particular English Language Learners (ELLs), communicate about mathematics.

While there have been numerous studies on mathematics discourse in the classroom, there has been limited research on secondary students’ discourse (Huang, Normandia, & Greer, 2005). Furthermore, Huang et al. (2005) noted research at the secondary level is typically focused on communication as a means of acquiring mathematics content rather than gaining an understanding of it. They suggested there is a correlation between communication and understanding, and argued teachers should be explicit in the use of reasoning in the classroom because students do not automatically pick up on the nuances of mathematical discourse from the teacher or classroom discussion. Therefore, systematic integration of mathematical thinking and talking into the mathematics curriculum would greatly benefit mathematics education (Huang et al., 2005).

Ideally, an increase in discourse between students and teachers helps students "gain a greater understanding of…mathematics and become better problem solvers" (Brenner, 1998, p. 154). In an analysis of several articles, Janzen (2008) found that teachers can help ELLs by paying attention to how they interact with each other and allowing students to talk about and explain their reasoning as they work through problems. When students participate in mathematical discourse, they are able to find alternative ways of solving problems that also helps the teacher understand what the students know and what gaps they have in their understanding (Janzen, 2008).

Mathematical discourse that supports students' learning, is complex and presents new challenges for teachers and students, especially ELLs (Brenner, 1998; Sherin, 2002). Although it is challenging, developing this type of classroom contributes to the development and learning of students and teachers (Sherin, 2002). Research suggests that starting students, particularly ELLs, in small groups, leads to increased student participation in large group discussions (Brenner, 1998). In a study of two classrooms using the same curriculum, Brenner (1998) found students who were given explicit guidance on how to participate in small group discussions participated more actively in whole-class discussions.

Perssinni and Knuth (1998) suggested discourse plays a vital role in the mathematics classroom and should be fostered so that students treat the speech of their classmates as "thinking mechanisms to be questioned and extended" (pp. 107-108). In a study of a fifth grade Japanese classroom, Wertsch and Toma (1995) found that discourse in the classroom was either used to deliver accurate information between the speaker and audience or to invoke thought and discussion. Cobb and Bauersfeld (1995), defined dialogue that invokes thought and discussion as inquiry mathematics. Knuth and Peressini (2001) described these functions of discourse as being either univocal or dialogic. Univocal discourse is used to convey exact meaning, such as when one participant speaks and the other one listens; and dialogic discourse is defined as discourse that takes place between participants, where one participant initiates the discourse and a dialogue ensues (Knuth & Peressini, 2001). Reform-based classrooms tend to involve more dialogic discourse than univocal.

English Language Learners (ELLs) Participation in the Mathematics Discourse

When referring to students as ELLs, I mean "students who participate in multiple-language communities" (Moschkovich, 2002, p. 198). ELLs present their own set of challenges for teachers striving to develop more mathematics based discourse in the classroom. However, when working with ELLs, teachers should not limit their consideration to the obstacles these students face, they also need to consider the resources they bring to the classroom (Moll et al., 1992).

In addition to supporting native English speaking students, classroom instruction needs to support "bilingual students' engagement in conversations about mathematics that go beyond the translation of vocabulary and involve students in communicating about mathematical concepts" (Moschkovich, 2002, p. 208). It is important for students to be afforded the opportunity of using mathematical language, in particular "to pose mathematical questions, describe the solutions to problems, explain and justify their solutions, present arguments for or against conjectures, and defend their generalizations"

(Moschkovich, 1999, p. 10). ELLs are supported in this goal when they are provided opportunities to participate in mathematical discussions that are rich in mathematical vocabulary and discourse (Moschkovich, 1999 & 2002).

Teachers can support ELLs acquisition of English by using instruction that is rich in mathematical vocabulary (Moschkovich, 2012). Language experts suggest that acquisition of vocabulary in a second language is most successful when they are repeatedly exposed to the vocabulary and actively involved in using the language, for example, consistently hearing and using mathematical terms and vocabulary in a variety of ways throughout the year (Moschkovich, 2012). In their study of two elementary school classrooms, Khisty and Chval (2002) found students needed to hear appropriate mathematics before they could use it. They observed that in the beginning of the year students used few words when responding to the teacher's questions and/or each other's comments; towards the end of the year they were speaking in complete sentences and using correct mathematical vocabulary. This transformation came about because their teacher, Ms. Martinez, modeled the mathematics vocabulary for them and demonstrated by her behavior that she expected the students to participate. This follows with Sherin's (2002) observation that when there is an expectation for students to share their ideas they are more likely to do so.

Working on Challenging Math Problems Students Construct Their Own Knowledge

As classrooms become more diverse, the role of the mathematics teacher is changing. It is no longer sufficient for teachers to stand in front of the classroom dispensing knowledge while students sit at their desks listening, taking notes, and completing worksheets. In reform oriented classrooms teachers support the development of students' cognitive knowledge by presenting real-world problems that allow students to actively grapple with mathematical problems and construct their own knowledge (Stein, Engle, Smith, & Hughes, 2008). Stein et al. (2008) described this classroom setting as one that starts with the teacher presenting a real-life, challenging math problem, allowing the students to explore the problem while working on finding a solution, and concluding the lesson with a whole-class discussion that involves students presenting and justifying their solutions. During the process of exploring and solving the problem, students are encouraged to work together, in pairs or small groups; afterwards the teacher guides the students in a whole-class discussions that leads to an understanding of the mathematical concept being studied.

However, this change comes with a price. Reform-oriented mathematics classrooms place new demands on teachers and students, especially ELLs. For example, students work on open problems that can be solved in different ways, and are expected to discuss their solutions in pairs, small groups, and/or with the whole class. Additionally, there is an expectation they will ask good questions, rephrase problems, provide good explanations for their solutions, be logical in the solutions, justify their work, and consider the answers of others (Boaler & Staples, 2008). In a five year longitudinal study of three high schools, Boaler and Staples (2008) found that Railside, the most ethnically and linguistically diverse of the three schools, was the most successful in closing the achievement gap among the students. They found teachers were successful because they valued the multi-dimensionality of mathematics. At Railside, the multidimensional nature of the mathematics classes led to an increased level of student success because students knew there was more than one way to be successful in mathematics and they could excel at some of them (Boaler & Staples, 2008).

Students' Mathematical Communication in Small Groups

Brenner (1998) suggested small group discussion can help ELLs build confidence in their mathematics discourse. Teachers should be aware of the fact that small group discussion needs to be developed among their students. For ELLs whole-class discussion can be somewhat challenging if not daunting. In her observations of two mathematics classrooms, Brenner (1998) noted that the teacher who used small group discussions effectively had more student participation in whole-class discussions that were rich in mathematical content.

Another benefit of small groups is that ELLs are able to enjoy and participate in social interactions with other students (Brenner, 1998). ELLs should also be given the opportunity to participate in large group instruction, because they are able to gain access to mathematical terms and vocabulary they may not get working with their "peers who are also developing their second language skills" (Brenner, 1998, p. 158). Student participation in group discussion benefits both the students and the teachers by providing students the opportunity to achieve new goals that are in line with reform mathematics and allowing teachers to assess students' understanding of the lesson content (Brenner, 1998).

Perhaps, small group discussion provides the greatest benefit to ELLs because it provides them the opportunity to discuss mathematics in a smaller setting prior to discussing it in a larger group. Brenner (1998) suggested language minority students often have "more sophisticated conversation" (p. 170) when working in small groups that can eventually lead to similar discussions in larger groups. When denied the opportunity to converse in small groups, ELLs are less likely to participate in large group discourse.

Boaler and Staples (2008) found that when students are held responsible for each other's learning they tend to work more effectively in groups. Blunk (1998) described how one teacher, Lampert, worked with her students to establish effective group work habits. In the beginning of the year Lampert explicitly told her students what small groups were and why they would be working in groups. Furthermore, she talked about her expectations; in particular, their responsibility for their own behavior, how they were to help each other, and how they could only ask for the teachers help when everyone had the same question (Blunk, 1998). Throughout the year, Lampert continued to support the students' work in small groups by recognizing them when they were working well as a group and correcting groups when they were not working well together. Students in Lampert's class became successful working collaboratively in small groups because she taught them and they understood what was expected (Blunk, 1998).

Types and Level of Teacher Questions Shape the Mathematics Classroom

The types and level of questions teachers ask shapes the mathematics classroom (Boaler & Brodie, 2004), impacts student engagement (Kazemi & Stipek, 2001), and influences student learning (Purdum-Cassidy, et al, 2014). Effective teacher questions lead to deeper mathematical thinking and reasoning within students (Purdum-Cassidy, et al, 2014) and when teachers ask important questions, students ask important questions (Boaler & Brodie, 2004). Therefore, if teachers want their students to ask more conceptual, probing, or exploring type questions, then teachers need to ask more conceptual, probing, and exploring type questions; if they want students to extend their thinking, teachers need to ask questions that extend students' thinking (Boaler & Brodie, 2004).

In a reform-oriented classroom, questions are used to elicit information about students' thinking, encourage dialogue, and help students build conceptual knowledge (Purdum-Cassidy, et al, 2014). It is important for teachers of mathematics to develop a repertoire of questions that push students' cognitive development of mathematics and hold them accountable for rigorous, disciplined ways of communicating their thinking (Smith & Stein, 2011). Questions that elicit high expectations of conceptual thinking in students; permits mathematics to drive student engagement (Kazemi & Stipek, 2001), helps students discover how to reason mathematically, and realize they can make sense of mathematics (Smith & Stein, 2011).

Effective teacher questions are powerful instructional tools and have been recognized as a "critical and challenging part of teachers' work" (Boaler & Brodie, 2004, p. 774). The questions teachers ask play an important role in guiding students as they navigate "the mathematical terrain of lessons" (Boaler & Brodie, 2004, p. 781). It is easy for teachers to ask students to describe their strategy for solving a problem; "it is more challenging…to engage students in genuine mathematical inquiry and push them to go beyond what might come easily for them" (Kazemi & Stipek, 2001, p. 123).

In a study of three separate schools, Boaler and Brodie (2004) developed a method of coding teacher questions to inform their study. They developed nine categories of teacher questions that came from a study of different examples of teacher instruction (Boaler & Brodie, 2004). The nine categories were: 1) gathering information, leading students through a method, 2) inserting terminology, 3) exploring mathematical meanings and/or relationships, 4) probing, getting students to explain their thinking, 5) generating discussion, 6) linking and applying, 7) extending thinking, 8) orienting and focusing, and 9) establishing context (Boaler & Brodie, 2004, p. 777). Teachers can use these types of questions during instruction to extend student thinking and deepen their conceptual knowledge of mathematics.

Gestures Convey More than Words Alone Can

Not only do gestures help convey the meaning of a speaker's word, but gesturing reduces the demands placed on the "speaker's cognitive resources" (Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001). Gestures can be a reflection of a student's problem solving strategies as well as completing the picture of their verbal explanations (Bjuland, Cestari, & Borgersen, 2008; Goldin-Meadow, et al, 2001). According to Radford, Berdini and Sabena (2007), gestures provide students with a "visual and sensory-motor representation" (p. 526) of their explanations and helps them imagine it. Furthermore, gesturing helps students visualize what they are talking about, reduces the cognitive load of explaining, improves memory, and helps students to organize information (Goldin-Meadow, et al, 2001; Radford, et al, 2007).

Through the use of purposeful gestures, teachers can give meaningful instruction (Yoon, Thomas, & Dreyfus, 2011). Goldin-Meadow, Kim, and Singer (1999) suggested that gestures can guide the students' attention to certain aspects of a problem or present another representation of a concept. For example, during a lesson on perimeters, a teacher could describe what the perimeter is while tracing the outside of a rectangle with her finger. Yoon, Thomas, and Dreyfus (2011) suggested that teachers use the following two strategies to help students in their study of mathematics: first, provide students with the space necessary to use gestures, and second increase students' awareness of how to use them to elaborate on their thinking. By using gestures in their instruction, teachers can support students' learning and help them develop the use of gestures in their own communication.

CHAPTER III

METHOD

Data Collection

The data for this study came from research conducted by Heather Johnson, a professor and researcher at University of Colorado Denver. As part of my own study, I assisted in the collection of data. The study was conducted in an urban middle school in the western United States and consisted of six lessons taught to four sections of 7^{th} grade students. The lessons used task situations involving a changing rectangle to support students' reasoning about covarying quantities (Johnson, 2013). From the four sections, 14 students were chosen to participate in a follow up interview with Johnson. The students were chosen because they frequently communicated their mathematical ideas during whole group lessons in the classroom.

Students were paired with another student from their section. When more than two students were from the same section, Johnson consulted with the teacher to determine which students would work best together. The student pairs were Sergio & Rico (S&R), Tomas & Simon (T&S), Jorge & Tien (J&T), Navarro & Daria (N&D), Blanca & Crista (B&C), Myra & Elena (M&E), and Olivia & Sierra (O&S). The names of the students, used in this study, are pseudonyms.

Interviews were conducted with pairs of students at the school and were recorded. At this school, 46% of the student population had never exited the ELL program (i.e., they are not proficient in English), 18% had been re-designated from one level to another (or exited ELL), and 36% have never been in ELL.

During the interviews the students were given two separate tasks. The first task was a continuation of the lessons taught during the sections, and examined the relationship between the increasing height of a rectangle and the volume of the rectangle (Figure 1).

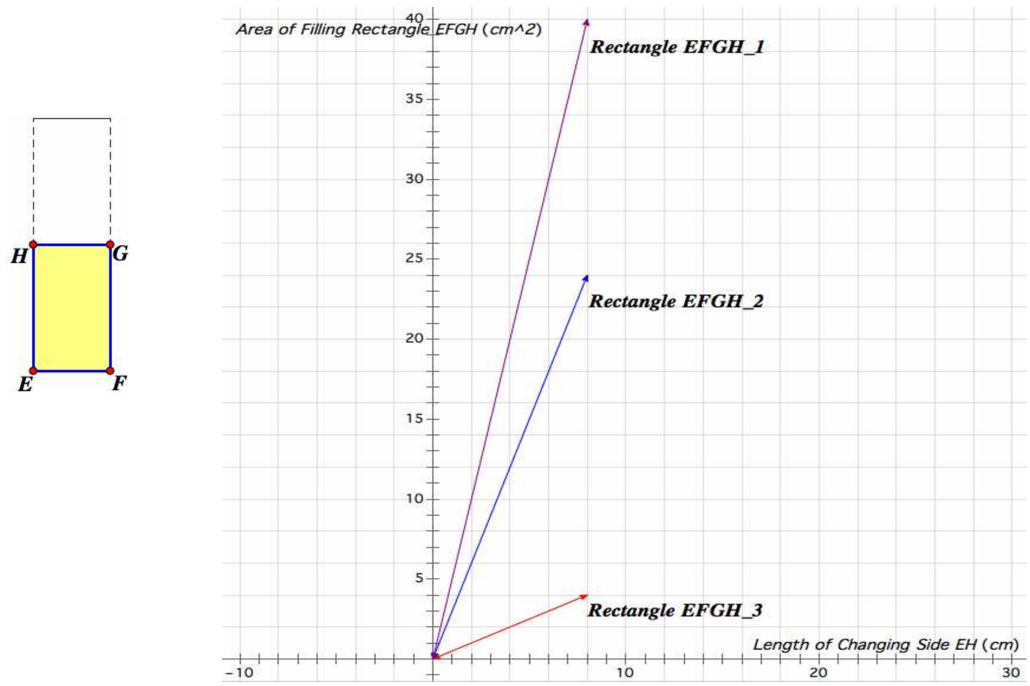

Figure 1. Areas of a filling rectangle

During the first task, students were asked to explain how the area of rectangle EFGH changed as the length of side EH increased. For the second task students were asked to compare the relationship between the area of ABCD and the length of side AD (Figure 2). For more details regarding the tasks see Johnson (2013).

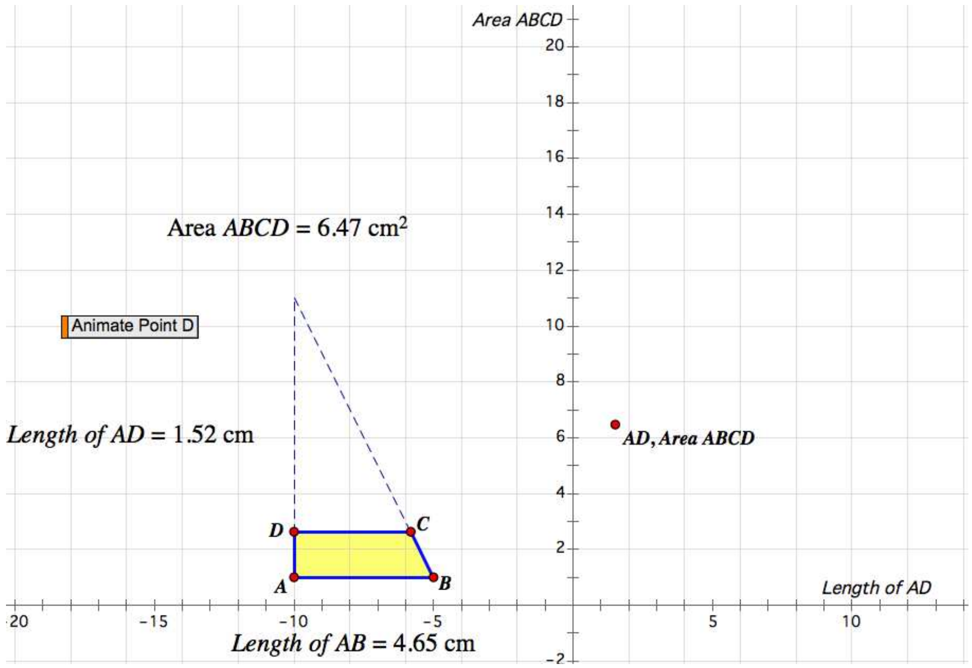

Figure 2. Areas of a filling triangle

Data Analysis

Following the interviews, the video recordings were transcribed by myself and the researcher. To ensure credibility and accuracy of the transcriptions, each transcript was reviewed by an additional researcher (Patton, 1999).

For the purposes of this study, I reviewed the video recordings and transcripts with the intent of examining the questions the researcher asked and how the students responded to the researcher and each other. Additionally, I made note of how the students used gestures, mathematical representations, manipulatives, and language to communicate about quantity and covariation, in particular the relationship between the side length of AD and the area of ABCD, and how they supported and justified their understanding. Even though I reviewed both the rectangle and triangle recordings and transcripts, I chose to focus on and use the results of the triangle interviews because it was a new problem for the students.

After an initial review of the transcripts and videos, I developed four types of questions that I felt were used in the interviews and discussed them with Johnson. The four question types were; 1) an overarching question meant to focus students on quantity and covariation, 2) a question related to the overarching question, but more specific, 3) a question that engaged the other student in the conversation, and 4) a question that redirected the conversation. These types were chosen based on the anticipation that these would be the types of questions to come up during the interviews. Table 1 shows examples of the four question types. In the discussion that follows I will describe how I coded questions as type 1, 2, 3, or 4.

Table 1. Examples of four question types

Question type	Example
1) Overarching question	Researcher: …go ahead and press animate point, and then talk to me about what changes and what stays the same. (Jorge & Tien) Researcher: can you talk to me about how the area of ABCD changes as AD increases? (Navarro & Daria) Researcher: …if you were going to create a graph…that would relate the side length of AD, and the area of ABCD, for all the lengths of AD…what would it be like? (Olivia & Sierra)
2) Question related to an overarching question, but more specific	Researcher: How does it (the area of ABCD) increase as AD gets bigger? What's the increase like? (Myra & Elena) Researcher: What two things would it relate?...if you plotted points on this graph...how would that relate to the triangle? (Blanca & Crista)
3) Question to engage another student in the conversation	Researcher: What do you think Rico, does that sound about right? (after Sergio stated the area of ABCD "kind of decreases because not that much space can be inside of it.") (Sergio & Rico) Researcher: What do you notice Tien, about what changes and what stays the same? (Jorge & Tien)
4) Question to redirect the conversation	Researcher: …you're trying to find a formula for ABCD? I'm interested, can you talk to me, how does the area change as the side length goes up? (Navarro & Daria)

After a quick review of the transcripts, Johnson and I decided that I should start by reviewing and highlighting the rectangle transcript of Myra & Elena, because not only did they talk with the researcher, but they talked with each other as well. The review consisted of highlighting each question the researcher asked and determining where each question group started and ended.

In deciding what counted as a question, I followed a pattern similar to Boaler & Brodie (2004). In particular, I did not include utterances in the form of a question (for example, "would you like to see the graph?") as questions. However, utterances that were meant to solicit an answer (for example "then talk to me about what changes and what stays the same") were counted as a question.

Initially, I looked at the questions as groups. A group consisted of questions that were related to each other either because they were related to the same overarching question, followed a students' chain of thought, or refocused the students' thinking. For example, in the following dialogue the question group starts with the researcher asking "what changes and what stays the same?" A new question group starts with the researcher asking about a graph for the filling rectangle.

Researcher: What changes and what stays the same? *[Start of a question group]*

Elena: Okay, what changes is the length.

Myra: Of EH and the area of EFGH and what stays the same is the length of EF.

Elena: F, yeah the base.

Researcher: Okay, what if we change the side length of EF, like make it really small.

Elena: Then EF still says the same but the, um, area *[Sweeps palm of right hand in front of computer screen.]* is smaller, because each of the side lengths, each of the, H and E side length gets bigger and EF stays the same.

Researcher: And, what if I make EF really big?

Myra: Um, the length of EF will change and how Elena said,

Researcher: Wait, the length of EF? Did you mean the length of EH?

Myra: EF would change *[Moves pencil back and forth along EF shown on the computer screen.]*, I mean would not change, and the length of EH will *[Emphasizes 'will' when speaking]* change, and also the area will change. It may get bigger or smaller.

Start of new question group

Researcher: And if I were to press the show graph so that graphs would appear, and if you would compare the graph of when EF is really long to when EF is really short, do you have any sense of how the show graphs would compare?

Like in the example above, new groups often started when Johnson asked the students an overarching (type 1) question. Occasionally, a new group started when the students were asked a question that redirected the conversation (type 4).

After highlighting the questions and identifying question groups, I started coding the questions by type. In deciding how to classify a question, first I asked myself if it was an overarching question, a derivative thereof, or related to an overarching question, but more specific.

The overarching questions (type 1) were based on the researcher's interview questions. Type 1 questions were one of the following or a derivative thereof; a) what do you anticipate will happen when you click animate point, b) how is the area of ABCD changing as the length of AD is increasing, or c) suppose you created a graph relating area ABCD and the length of AD, what do you expect the graph to be like. For example, one of the questions asked of Olivia & Sierra, "as the length of AD gets larger, how does the area of ABCD change?" was coded a type 1, because it was a derivative of the overarching question “how is the area of ABCD changing as the length of AD is increasing?”

Questions related to the overarching question, but more specific (type 2), were also based on Johnson’s interview questions, but contained more detail. For example, she asked Myra (referring to the aforementioned overarching question), “Could you just describe how the area gets bigger? Like, does it get bigger just like the area of the rectangle got bigger?” was coded type 2 because the researcher was more specific by relating it to the rectangle problem.

Questions were coded type 3 when the researcher asked a question of a specific student and the dialogue prior to the question showed that the student had not been participating in the discussion. For example, at the start of her interview with Jorge & Tien, Jorge answered the questions and Tien made only a small remark until the researcher asked what she is thinking.

Researcher:	Instead of a filling rectangle, it's going to be a filling triangle. And so what I'd like one of you to do is just go ahead and press animate point, and then talk to me about what changes and what stays the same. [*Jorge presses touchpad,*] And then anywhere on the touch pad.
Tien:	Oh
Jorge:	I wanna say that these two points are getting smaller as it goes up to like an actual triangle.
Researcher:	And which points are these two again? You can use the letters for them, just so I'm sure I know which ones you mean.
Jorge:	I can’t see it. [*Researcher moves computer closer.*] D and C
Researcher:	Okay
Jorge:	They're going to get like, well, I can see as you animate the point, they're going to get smaller and smaller. It gets smaller as it gets to the point.

Researcher: *What do you notice Tien about what changes and what stays the same?*

I coded the last question in this example as a type 3, because the researcher specifically asked Tien a question and she had not been participating in the dialogue.

A type 4 question was different from a type 3 in that generally, either one or both of, the students' attention had drifted from the current objective and the researcher asked a question to bring the discussion back to the intended topic. For example, in the excerpt below Crista and Blanca are trying to calculate the area of ABCD, then the researcher attempts to redirect their attention with the final question.

Crista: *[continues to calculate the area of ABCD]*

Blanca: So see how that's *[points to AB]* like the same length as right there *[points to DC]*? Um, I think it stays the same, cuz no matter what you're going to have that same length, but, you're gon – , you're gonna' divide it by two.

Researcher: Ok. So is shape ABCD, is *that* a triangle? *[Type 4 question]*

As I analyzed the transcripts I kept a record of the question type in a table, which included a column for each student pair, the type of question asked, follow up questions and notes.

Following my review of Myra & Elena's rectangle transcript, I reviewed, highlighted, grouped, and coded questions in the rectangle transcripts of Tomas & Simon, Jorge & Tien, Blanca & Crista, and Navarro & Daria respectively. At this point I left my work with the rectangle transcripts and began highlighting and coding the triangle transcripts. After completing the triangle transcripts, I went back to highlighting and coding the rectangle transcripts of Olivia & Sierra, and Sergio & Rico.

While working with the triangle transcripts, I found several instances where the questions did not fit any of the four types. In particular, the researcher was probing students for an explanation of their responses and thinking (Boaler & Brodie, 2004; Smith & Stein, 2011). Therefore, I added a fifth type (table 2), a question that was meant to push the students thinking and explanation. Notice in the following example how the researcher follows Sierra's response with a question that is pushing her to elaborate on her thinking (type 5).

Researcher: What do you think Sierra? How does the area increase? Is it like the rectangle? If you had to describe it, what's the increase like?

Sierra: I don't think it's going to be like the rectangle.

Researcher: Okay. Why not? *[Type 5 question]*

Table 2. Examples of type 5 questions

Question type	Example
5) Question that pushes the students to elaborate on their response	Researcher: Why does that make sense? (Blanca & Crista) Researcher: Okay so can you help me understand that? Can you show me the graph? (Myra & Elena) Researcher: How does it increase? Can you describe it? (Sergio & Rico)

During my analysis of the transcripts, I noticed students using gestures, such as moving their hands and pointing to the computer screen or handout, as they responded to questions, and explained their answers. This led me to ask questions about the gestures, such as how often were students using gestures, were all the students using them, how were they being used, etc. (Corbin & Strauss, 2008). Based on my analysis of the video recordings and transcripts, I identified three types of gestures (Table 3). In particular; a) pointing at the screen, paper, or etc.; b) illustrating a motion such as pinching fingers together to represent the filling of a triangle, raising arm at an angle, or etc.; and c) nodding in agreement.

Table 3. Examples of first three gestures types

Gesture Type	Example
a) Pointing	*Crista points to the rectangle on computer and swirls pencil around perimeter *Jorge points to his paper with his pencil while saying "It's going to tell us the length of A and D *Tien points to endpoint of graph on the computer screen, then the calculation area for ABCD, then back to the endpoint on the graph *Tomas points to the animation until triangle is filled, moves finger away when triangle is filled
b) Motioning with fingers or hands	*Elena holds palms about 10 inches apart and parallel to each other *Navarro creates "box" by placing hands together with thumb and index finger outstretched *Olivia presses palms together vertically, then moves arms up and down *Sergio holds left hand near laptop screen, cups hand and rotates hand back and forth starting clockwise like he is turning a dial *Sierra raises hands up and together to form triangle between thumbs and index finger, then moves index fingers down to thumbs
c) Nodding	*Nods head in agreement

As I continued coding the transcripts, I noticed gestures that did not fit the three aforementioned types, in particular tracing a sketch or graph and pressing the “animate point” button. Hence, I added two more types of gestures (Table 4): d) using manipulatives (such as wikki stix), tracing an object, sketching, or drawing and e) others.

Table 4. Examples of type d and e gestures

Gesture Type	Example
d) Using manipulative, tracing, sketching, or drawing	*Daria uses pencil to follow along arc she drew on her paper *Elena places wikki stix at the origin, wiggling up (stair step) at approximately 45 degree angle *Jorge draws line segment on coordinate plane, beginning at origin *Navarro follows finger up along left side of triangle on graphing sheet, down the slanted side, and back up the left side *Simon traces finger along vertical side of rectangle *Tien moves point D on the computer to near the midpoint of AD, drags point D back and forth while saying "it's moving a lot."
e) Other	*Blanca presses show graph and then presses animate point *Daria places hand on head and starts spinning pencil in her hand *Myra slides graphing paper to the right, peeks at the back of the other paper in front of her and flips it over so the blank side is up *Rico raises right hand

After coding the transcripts, I began examining the data for connections between the researcher's questions and student responses. I used constant comparison analysis (Corbin & Strauss, 2008) to look for similarities and differences. For example, I noticed when Johnson asked the overarching question "how is the area of ABCD changing as the length of Ad is increasing?" each of the student pairs in the same way (a similarity), they said it was increasing. Then when she followed up with "how does the area increase?" their responses became diverse (a difference). Olivia, for example, did not know how to respond while Sergio said it will go up "by a small amount."

To address the three research questions, interview recordings and transcripts were analyzed by comparing the types of questions asked by the researcher to the student participation, and gestures. Additionally, I examined how often students communicated directly with the researcher (student-to-researcher) and compared it to their communication with each other (student-to-student). My intent in analyzing the student-to-researcher (S-R) and student-to-student (S-S) discourse was to consider whether or not S-S discourse served as a resource in student discourse. The data from the interviews and transcripts were reviewed multiple times to find common themes and patterns in the discourse between the students and researcher. The results of my analysis follow in the next chapter.

CHAPTER IV

RESULTS

Questions Guide Student Participation

In the interviews, Johnson began with overarching questions then guided students' participation with effective follow up questions. Table 5 shows the five types of questions, how many of each type were asked of each student pair, and the percentage of each. The totals show how many questions student pairs were asked and how many questions of each type Johnson asked overall.

Table 5. Number and percentage of question types asked each student pair

	Type 1		Type 2		Type 3		Type 4		Type 5		Total	
	#	%	#	%	#	%	#	%	#	%	#	%
Blanca & Crista	4	16.7%	8	33.3%	1	4.2%	5	20.8%	6	25.0%	24	9.7%
Jorge & Tien	4	8.9%	10	22.2%	6	13.3%	5	11.1%	20	44.4%	45	18.1%
Myra & Elena	3	6.5%	10	21.7%	2	4.3%	6	13.0%	25	54.3%	46	18.5%
Navarro & Daria	3	10.3%	12	41.4%	3	10.3%	4	13.8%	7	24.1%	29	11.7%
Olivia & Sierra	4	7.5%	21	39.6%	4	7.5%	1	1.9%	23	43.4%	53	21.4%
Sergio & Rico	4	14.8%	11	40.7%	4	14.8%	2	7.4%	6	22.2%	27	10.9%
Tomas & Simon	4	16.7%	2	8.3%	3	12.5%	0	0.0%	15	62.5%	24	9.7%
Total	26	10.5%	74	29.8%	23	9.3%	23	9.3%	102	41.1%	248	

In the interviews Johnson asked 102 type 5 questions (table 5). That equates to 41% of the questions (table 5) being higher order questions that encourage students to think more deeply and explain their thinking in greater detail. The type 1 and type 2 questions guided student thinking and participation and accounted for another 40% of the questions. Lastly, type 3 and type 4 questions directed the student discourse and accounted for 19% of the questions.

Using type 5 questions, not only was Johnson able to push students to elaborate on their thinking but she was also able to help students be specific in their responses. Notice in the following example how Johnson uses a question to get the student to specify what “it” is.

Researcher: And now let’s imagine how the area increases as the length of AD increases. So maybe let’s just start with a question here. If you press animate point, *can you tell me how the area of ABCD changes as AD increases?*

Simon: It increases, but as it gets bigger and bigger then it starts increasing more slowly because there’s not as much um, room I guess.

Tomas: Like that *[Points to the white space near the top of the triangle]* it goes like that and then it starts slowing down.

Researcher: *What does it mean to increase more slowly?*

Simon: Um, that it has a smaller rate of change, the rect- the go up, ah—as it gets bigger because uh, when you press animate point it goes like, wait for it still.

Tomas: It goes slowly right there. *[Points to the triangle as it is reaching the filled amount.]*

Simon: It goes one, two, three, four, five, six, seven-, eight— nine— ten—, and then it just starts to increase, increasing by less each time.

Researcher: *And when you’re saying it’s the it—it goes slowly. Can you tell me what you’re looking at? What’s it? [type 5 question]*

Simon: The area.

In the above dialogue, Johnson used the type 5 question to push Simon to be specific and use mathematical language in sharing his thinking about the increase in area ABCD. Even if she understood what he meant by “it,” she expected him to use mathematical language to specify that “it” meant “the area.” This example demonstrates one of the ways teachers can use questions to push students to use mathematical language and be specific in the thinking.

Throughout the interviews Johnson used student responses to guide her use of follow on questions during the interview and adjust her questions for the following interviews. For example, in the dialogue below Johnson asked Sergio & Rico, the first pair to be interviewed, what they expected to happen when she pressed animate point (type 1). Rico responded first by trying to remember how to find the area of a triangle, then he identified the shape as a trapezoid. Following his response, the researcher rephrased the question and asked, "as the length of AD changed, what do you think would happen to the area?"

Researcher: If I click on the show rectangle, you can see how we are just filling this part of the rectangle, rather than the whole rectangle like we were filling before. So if I press animate point, what do you expect is going to happen?

Rico: Um, as I know with a triangle, you have to do, um wait don't you have to do, I'm not sure, base times height divided by two. And that looks like a trapezoid and if that was increasing, you'd have to do base one plus base two divided by two or times two I'm not sure, so this would be more complicated.

Researcher So if we were actually finding the amount of area…it would be more complicated. And I'm thinking, let's not worry so much how we would actually find it using a formula. But if I press animate point…that's going to change the length of AD, and so as the length of AD changed, what do you think would happen to the area?

Sergio: You know, it will increase, but, well, by a small amount.

In the rest of the interviews, the researcher asked each pair about what would change and what would stay the same, as can be seen in the following excerpt from the second pair to be interviewed, Tomas & Simon.

Researcher: Okay, I'm going to have us look at a situation that is no longer a filling rectangle. So instead of a filling rectangle we are going to have a filling triangle. And so what I want one of you to do is just press animate point. And, *let's talk about what changes, what stays the same*.

Simon: Um, the area changes and the length of AD changes and uh, but uh, I guess nothing technically stays the same because AB also changes and I guess because the right triangle so does the hypotenuse.

Researcher: What do you think Tomas?

Tomas: I notice that line segment DC changes. It becomes more narrower, more narrow, as it gets to the top. The area has to change, the area because it's not constant, it's not one equal shape, so that would change the area.

By changing the wording of this question the students' responses became less procedural in nature and more conceptual. For example, in the excerpts above Rico's initial response was procedural (he wanted to calculate the area of ABCD); whereas, Simon and Tomas answered by noting the changes in the area and side lengths (conceptual).

Johnson used questions to redirect students' thinking (type 4) from procedural to conceptual thinking. In the following example, notice how Myra & Elena begin by explaining the relationship between the length AD and the area ABCD procedurally. Johnson uses a combination of type 2 and 4 questions that guide the students to delve further into a conceptual understanding of covariation.

Researcher: So, yeah, so if I press animate point, what's the increase in the area of ABCD like? *[type 2 question]*

Myra: Um—

Elena: What is the increase like?

Researcher: Yeah.

Elena: Well, you just multiply the length of AD [*points to Length of AD*] and the length [*pointing toward Length of AB*].

Researcher: Mhm. And if we—

Elena: Wait, doesn't it just increase by [*laughs*]—

Researcher: Well, and if we don't worry so much about, I purposely picked a shape that is really hard to calculate area, and I did that purposely, and if we don't worry so much about how you actually find the area, if you just think about—*[researcher uses an utterance to redirect the students' thinking]*

Myra: Base one plus base two times height divided by two.

Researcher: Mhm. and so if we just think about how does this area [*motions fingers over shaded region of filling triangle*] change? How does it increase as AD gets bigger? What's the increase like? *[researcher uses type 4 question first to redirect students' thinking, then a type 2, followed by another type 2 quesiton]*

Myra: Oh, I get you now! Uh [*laughs*], the, how does it increase? Like, it's getting smaller and smaller every time [*motions hands upward creating the top of a triangle*].

In the above example, Johnson was able to help the students' move from thinking procedurally about the problem to thinking about it in a deeper conceptual way. As Johnson continues to push Myra & Elena to explain their thinking, both students demonstrate attention to quantity and covariation as can be seen in the following excerpt.

Researcher: So when you say it's getting smaller and smaller every time, do you mean there's less—

Myra: The area. [*said over Researcher*]

Researcher: The area's getting smaller?

Myra: The area's getting…it's getting bigger-

Elena: Bigger, I think it's getting bigger. [*Said as Myra was talking*]

Myra: Yes, the area's getting bigger, but how much it increases, it's getting smaller.

Another consideration was how often each student responded to the researcher's questions and how the questions may have affected student participation. Table 6 shows the number of times each student replied to the researcher's questions. It also shows the percentage of the students' replies based on the number of questions Johnson asked each pair. Students are listed by pairs, the pair with the least responses is listed first.

Table 6. Number and percentage of student replies to researcher's questions

Student	Number of student replies	Percent of student replies	Total number of responses per pair	Partner
Rico	12	31.6%	38	Sergio
Sergio	26	68.4%	38	Rico
Simon	24	60.0%	40	Tomas
Tomas	16	40.0%	40	Simon
Daria	30	54.5%	55	Navarro
Navarro	25	45.5%	55	Daria
Jorge	33	52.4%	63	Tien
Tien	30	47.6%	63	Jorge
Olivia	35	50.0%	70	Sierra
Sierra	35	50.0%	70	Olivia
Blanca	51	52.6%	97	Crista
Crista	46	47.4%	97	Blanca
Elena	59	52.2%	113	Myra
Myra	54	47.8%	113	Elena

Interestingly, except for two pairs, Sergio & Rico, and Tomas & Simon, there was less than a 10% difference in the number of times student pairs replied to the researcher's questions. On average, one student, from each pair, responded 56.7% of the time and the other student responded to 43.3% of the questions.

The pairs with the lowest percentage of questions to engage another student (type 3), also had the smallest difference in their participation. On the other hand the pair with the largest difference in participation, Sergio & Rico, also had the highest percentage of type 3 questions (table 7). This led me to believe that there was a correlation between the level of participation and the number of type 3 questions the researcher asked. Upon further examination, I found that in five of the interviews the researcher asked more type 3 questions of the student who participated the most. Table 7 shows the percentage of student participation (percent of responses) and the number of type 3 questions that researcher asked the pair (percent of type 3 questions). The last column of the table shows how many type 3 questions the student was asked. The students are listed by pairs, the pair with largest level of difference in percentage of responses is listed first.

Table 7. Percentage of student responses vs. number of type 3 questions researcher asked

Student	Partner	Percent of Responses	Percent of type 3 questions	Number of type 3 questions
Sergio	Rico	68%	14.3%	3
Rico	Sergio	32%	14.3%	1
Tomas	Simon	60%	12.0%	3
Simon	Tomas	40%	12.0%	0
Jorge	Tien	52%	13.0%	4
Tien	Jorge	47%	13.0%	2
Navarro	Daria	45%	10.0%	0
Daria	Navarro	55%	10.0%	3
Blanca	Crista	53%	4.0%	1
Crista	Blanca	47%	4.0%	0
Myra	Elena	48%	4.3%	2
Elena	Myra	52%	4.3%	0
Olivia	Sierra	50%	7.4%	1
Sierra	Olivia	50%	7.4%	3

Further analysis of the type 3 questions and student responses shows a correlation between how many type 3 questions students were asked and how often they responded. Johnson used type 3 questions to directly engage a student in conversation. Students asked three or four type 3 questions responded more frequently (a mean of 55 responses), than did students asked zero, one or two type 3 questions (a mean of 46 responses).

Figure 3 shows the number of type 3 questions students were asked as it related to the number of times the student responded during the interview. Students who were asked zero type 3 questions responded an average of 46 times.

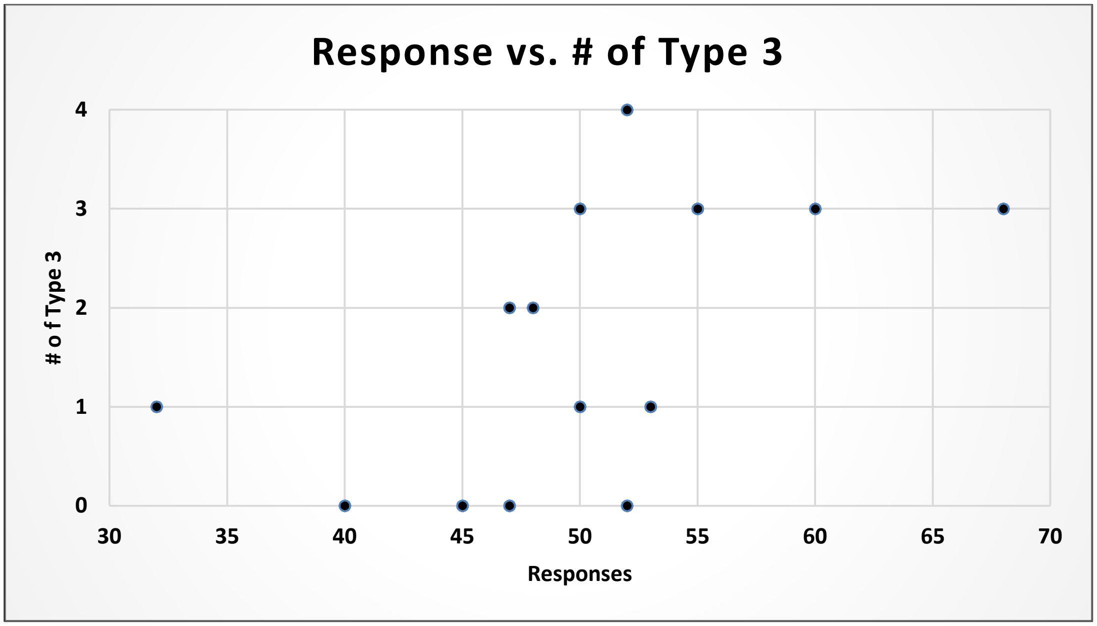

Figure 3. Number of student responses vs. number of type 3 questions

I found that Johnson's use of questions, in particular type 1, 2, and 5 questions, effectively assisted students as they navigated through a mathematical task that was new to them. She used type 1 and 2 questions to set the stage and help students explore this new concept. Furthermore, type 5 questions were utilized to probe students' conceptual knowledge and solidify their understanding. Through the effective use of questioning Johnson was able to guide student responses, help them stay focused on the objective, and elicit a deeper level of thinking from them.

Student Use of Gestures

In the interviews, Johnson encouraged students to use manipulatives and gestures to explain their reasoning. For example, when Tien explained that the area of ABCD will grow just a little bit at a time, Johnson suggested that Tien use the computer animation to explain what she meant.

Each of the students in this study used gestures in some form. Overall Elena, Olivia, Tien, and Crista had the highest use of gestures, respectively. Table 8 shows how often each student used gestures during their interview. The total column shows how many gestures were recorded for each student and total row shows how often each gesture type was used. Students are listed from the fewest number of gestures used to the greatest.

Table 8. Student gestures by individuals

	a	b	c	d	e	total
Simon	6	1	0	4	5	16
Rico	6	8	2	0	4	20
Blanca	5	3	1	7	4	20
Sergio	5	10	5	2	4	26
Daria	6	3	0	9	10	28
Myra	12	5	0	7	8	32
Sierra	8	5	3	11	6	33
Jorge	10	14	0	7	3	34
Tomas	9	2	0	18	1	30
Navarro	19	14	2	5	6	46
Crista	29	1	2	16	5	53
Tien	21	24	3	7	3	58
Olivia	26	26	4	7	2	65
Elena	31	23	1	7	13	75
Total	194	139	23	107	79	542

Elena had the highest (75) recorded use of gestures, while Simon had the lowest with 16. Overall, the most frequently used gestures were pointing (type a), illustrating a motion such as pinching fingers or raising an arm at an angle (type b) were second, and using manipulatives, tracing an object, sketching, or drawing (type d) were third; nodding (type c) and others (type e) were used the least.

Student-to-Student vs. Student-to-Researcher Communication

I analyzed the amount of student-to-student (S-S) vs student-to-researcher (S-R) discourse. Table 9 shows the number of times students directed their conversation to each other (S-S) and how often they spoke to the researcher (S-R). The total column shows how often the students spoke during the interview, while the total row shows the number of times remarks were directed at the researcher (S-R) and how often they were directed to the other student (S-S). Students are listed in pairs.

Table 9. Student-researcher vs. student-student discourse

Student	S-R	S-S	Total
Blanca	46	1	47
Crista	42	1	43
Jorge	31	2	33
Tien	27	0	27
Myra	40	1	41
Elena	55	2	57
Navarro	34	3	37
Daria	22	4	26
Olivia	35	0	35
Sierra	35	0	35
Sergio	26	0	26
Rico	16	0	16
Tomas	15	2	17
Simon	24	1	25
Total	448	17	465

While Navarro & Daria had the most student to student discourse (11.1%), 5 of their S-S exchanges occurred in the first minute of the interview, they were arguing over who would press the “animate” button (Daria finally “won” by picking up Navarro’s hand and placing his finger on the touchpad). During their subsequent S-S communication they were seeking assistance from each other. In the first one, Navarro asked Daria “what happens?” and during the second one Daria turned to Navarro for reassurance that she had the correct formula for calculating the area of ABCD. Other, S-S exchanges typically involved one student explaining something to the other one or completing sentences. For example, when Myra was struggling to understand how the area of ABCD was changing, Elena compared it to pre-algebra when they "found the area of the whole shape" and then continued to explain finding the area for a triangle.

The vast majority or the discourse, 96.3%, was between a student and the researcher. Although it may not be the only reason, it is probable that the design of Johnson's study caused students to direct most of their conversation to the researcher. In particular, Johnson had designed the interviews to solicit individual students' reasoning related to quantity and covariation, therefore her questions elicited more S-R communication than S-S communication.

CHAPTER V

DISCUSSION

To promote students' mathematical reasoning, teachers are being encouraged to move classroom practices away from teaching computational accuracy to focusing on understanding "mathematical ideas, relations, and concepts" (Kazemi & Stipek, 2001, p. 59). This focus is challenging pedagogically and demands that teachers engage their students in mathematical inquiry that pushes students to "go beyond what might come easily to them" (Kazemi & Stipek, 2001, p. 59). Asking good questions is one of the ways teachers expand students reasoning and thinking, because it affords students the opportunity to take ownership of their own learning (Purdum-Cassidy, et al, 2014).

The questions teachers ask not only shape the mathematical terrain of the instruction but, guides students as they navigate through it (Boaler & Brodie, 2004). In comparing the question types used in this study to Boaler and Brodie's (2004) nine types of questions, I found a correlation between four of the types. In particular, the overarching questions (type 1) is closely related to Boaler and Brodie's (2004) type 3, "Exploring mathematical meanings and/or relationships" (p. 777). In the interviews, Johnson used type 1 questions to support students' exploration of covarying quantities involved in the task. This same thing is accomplished in the mathematics classroom when teachers design and use questions that cause students to explore the mathematical relationships and meanings of concepts being taught (Boaler & Brodie, 2004).

The type 2 questions in this study correlated to Boaler and Brodie's (2004) type 8 question, "Orienting and focusing" (p. 777), in that they were more specific than type 1, because they were more specific they helped students focus on key elements. For example, if students did not say anything about the area of ABCD or the length of AD changing, when asked the first overarching question “what changed and what stayed the same?”, Johnson was able to orient their focus by specifically asking about the area of ABCD or the length of AD. By focusing their attention on the specifics, students were able to make connections between the mathematical representation (Boaler & Brodie, 2004) and the concept of covariation.

Questions that engaged another student (type 3) related directly with Boaler and Brodie’s (2004) type 5 question, “generating discussion” (p. 777), in that it solicited contributions from the other student. Stein and Smith (2011) recommended using these types of questions in conjunction with other teacher moves (such as revoicing, restating another’s reasoning, or comparing their own thinking to someone else’s) to guide mathematical discourse and student understanding. Based on the results of this study, this type of question is more effective when used more than once with a student and suggests two to three questions to be most effective in generating student participation.

Type 5 questions pushed students to elaborate on their thinking through probing, and correlated to Boaler and Brodie’s (2004) type 4 question, “probing, getting students to explain their thinking” (p. 777). In this study Johnson used type 5 questions to get students to be explicit in their explanations, explain their thinking, and delve deeper into their understanding of covariation. For example, when Sergio used “it” for the area Johnson asked what “it” was and when students told Johnson that the area of ABCD was increasing more slowly, she asked them to explain what more slowly meant. Incorporating questions that press students to elaborate their thinking can be challenging, because a teacher usually has many options at this point and it is uncertain where the students will go with the question (Boaler & Brodie, 2004).

I did not find a correlation between the type 4 (a question that redirected the conversation) in this study to Boaler and Brodie's (2004) question types. The lack of correlation suggests that type 4 questions represent another type of question teacher's use in the classroom. One way Johnson utilized type 4 questions was to redirect the students from thinking procedurally (for instance, calculating the area of ABCD) to thinking conceptually (noticing the relationship between the increase in the length AD and the area of ABCD). As suggested by Johnson's use, type 4 questions may support students shift from procedural to conceptual thinking.

Limitations

My initial intent for this study was to examine the affect ELL status had on the level of student participation. However, the ELL status of each student was not available and limited the scope of this study to examining the student participation in general. The United States is becoming more diverse and so are its classrooms. More students are learning English at the same time they are studying mathematics. Therefore, understanding how ELLs communicate their comprehension through verbal and non-verbal means is becoming increasingly important to teachers.

Due to the nature of the research conducted for this study, I was unable to contrast the nature of student to researcher (S-R) vs. student to student (S-S) communication. As noted in the previous chapter the original research was designed to solicit more S-R communication; therefore, there was very little S-S communication. When the students did engage in S-S communication, it was to solicit support and/or reassurance.

Another limitation was the small sample size. While I was able to examine the discourse within this small group it may not be representative of the larger student population. Further study of student interactions among themselves and with teachers, how they use gestures, and the impact of teacher questions on student participation will benefit teachers and educators.

Implications

An issue for further research would be to study the correlation between a student's ELL status, teacher questions, and how students communicate in response to the questions. Research of this nature would serve to prepare teachers for the continuing increase in the diversity of the student population. Furthermore, as classrooms move towards reform-oriented teaching there will be a greater need to understand how questions affect ELLs learning.

Additionally, research on how gestures are affected by gender and culture would benefit educators. Questions for further research include, do students of one culture use gestures more than another? Is there a difference in how girls and boys use gestures? Does the way students are grouped change how they use gestures (i.e. girl-girl, girl-boy, or boy-boy)?

In Johnson's study the students tended to communicate more with her than with each other. Was this a result of the nature and design of her study? In this study, the students were sitting next to each other, facing the researcher, with the laptop between the students and Johnson. Does the way a teacher sets up a classroom impact the amount of student-to-teacher (S-T) vs. S-S discourse? Also, the questions were designed to elicit individual student responses, hence did this affect the amount of S-T vs. S-S discourse?

Does the nature and design of teacher questions change the way students interact with each other and the teacher?

Lastly, the potential link between the types of questions asked of students (e.g., questions to engage another student (type 3)) and their level of participation (e.g., number of responses), merits further study. Students in this study who were asked three or four type 3 questions responded more than students who were asked zero, one, or two type 3 questions. Continued study of the correlation between the number and type of questions asked to students and their subsequent participation could benefit mathematics teachers of mathematics. This study was conducted in a small group setting with two students. Might a similar correlation be present within a larger group of three or more students or with students participating in a whole-class discussion?

Concluding Remarks

The landscape of the mathematics classroom in the United States is changing, and a greater emphasis is being placed on students obtaining a conceptual understanding of mathematics (National Governors Association for Best Practices Council of Chief State School Officers, 2010). This transition, from simply learning procedures to exploring mathematical concepts, is challenging for both teachers and students. Questions and gestures play an important role in student interaction during classroom discourse, whether it be whole-class, small groups, or a pair of students.

In a small way this study captured how teacher questions can guide student's navigation of mathematical terrain that is new and unfamiliar. It showed how gestures can play an important role as students communicate their understanding of mathematics.

However, additional research is needed to understand how this reform in mathematics education is affecting the increasing number of diverse learners in the United States.

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