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DESIGNING TECHNOLOGY-RICH TASKS TO FOSTER SECONDARY STUDENTS' COVARIATIONAL REASONING

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Using design experiment methodology, I have engaged in multiple iterations of task design and implementation to investigate aspects of technology-rich tasks that might foster secondary students' smooth conceptions of variation, and thereby lead to students' more robust covariational reasoning. Task design principles include: incorporating measurable attributes that are not time, problematizing the determination of numerical amounts of change, and varying the representation of the same measurable attributes. Students working on such tasks demonstrated smooth conceptions of variation. Implications include the design of instructional tasks related to function.

Consider a situation involving a turning Ferris wheel. Now consider two quantities involved: the distance a car has traveled around the wheel relative to a starting point at the base of the wheel and the height of a car from the ground. As the Ferris wheel turns, how do the distance and height change together? An individual attending to the different ways in which both the distance and height are changing in relationship to each other would be engaging in covariational reasoning.

Covariational reasoning can provide a foundation for secondary students' learning of function (e.g., Confrey & Smith, 1995; Ellis, Özgür, Kulow, Williams, & Amidon, 2015; Smith & Confrey, 1994; Thompson & Carlson, in press). Although more recent definitions of function use a correspondence between sets (e.g., the Dirichlet-Bourbaki definition), scholars laying groundwork for function investigated and represented relationships between varying quantities (e.g., Smith & Confrey, 1994; Thompson & Carlson, in press). Given the historical significance of covariational reasoning in the development of function, instructional tasks should provide secondary students opportunities to engage in covariational reasoning (e.g., Smith & Confrey, 1994; Thompson & Carlson, in press).

Covariational reasoning entails individuals' conceptions (Piaget, 1970) of variation in measurable attributes of related objects (quantities). I contrast two conceptions of variation posited by Castillo-Garsow, Johnson, & Moore (2013)—*smooth* (envisioning change as progressing) and *chunky* (envisioning particular amounts of completed change). Castillo-Garsow et al. (2013) posited that smooth conceptions of variation are more powerful, arguing that envisioning change as progressing entails envisioning particular amounts of completed change, because at some point the progressing change must stop. Smooth conceptions of variation afford students' attention to variation in the intensity of change occurring in a single direction (e.g., as the distance increases, the height of a car on a Ferris wheel will increase slower, then more quickly) (Johnson, 2015a).

Researchers have developed tasks designed in part to investigate students' covariational reasoning in clinical interview settings (e.g., Confrey & Smith, 1995; Ellis et al., 2015; Johnson, 2015a; Saldanha & Thompson, 1998). Yet, even when tasks involved computer simulations depicting measurable attributes changing continually, Ellis et al. (2015) and Saldanha and Thompson (1998) found that secondary students using covariational reasoning demonstrated chunky conceptions of variation. Drawing from empirical research with secondary students, Castillo-Garsow et al. (2013) conjectured that chunky conceptions of variation do not serve as a cognitive root for smooth conceptions of variation. Building from Castillo-Garsow et al. (2013), I argue that opportunities for students to develop and/or demonstrate smooth conceptions of variation are both useful and needed.

Using design experiment methodology (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), I have engaged in multiple iterations of task design and implementation to investigate aspects of technology-rich tasks that might foster secondary students' smooth conceptions of variation, and thereby lead to students' more robust covariational reasoning. In this proposal, I provide a sample technology-rich task and task situation, task design principles, and empirical results. I conclude with brief discussion/implications. At ICME-13, I intend to elaborate further in each area.

TECHNOLOGY-RICH FERRIS WHEEL TASK AND TASK SITUATION

The technology-rich tasks I have designed are sets of problems involving measurable attributes relating to a given task situation—a common experience that can unite a set of tasks (e.g., riding on a Ferris wheel). The tasks include dynamic computer environments (DCEs), such as the one shown in Fig. 1, which links a dynamic animation of a Ferris wheel and graph. The Ferris wheel task situation incorporated a sequence of tasks, each following the task template in Table 1.

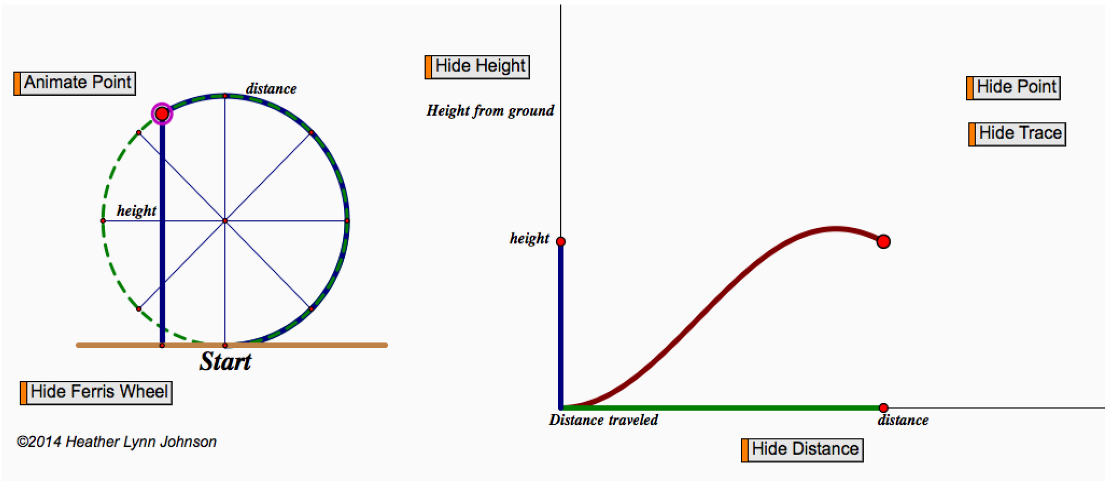

Figure 1: The Ferris wheel dynamic computer environment: Distance and height

Part	Task description
1	Identify changing attributes. Describe what changes and what stays the same.
2	Sketch a graph that represents a relationship between given attributes
3	Investigate how vertical and horizontal segments on graph relate to Ferris wheel
4	With Ferris wheel hidden and only graph segments showing, predict car's location
5	Compare graph sketched in Part 2 with trace shown on dynamic graph

Table 1: Ferris wheel task template

Innovative elements of the Ferris wheel DCE include: graphs with dynamic segments (See vertical and horizontal segments on graph in Fig. 1.) and the potential to hide/show animations and/or graphs (See action buttons in Fig. 1.). I designed the dynamic segments to allow students to predict, interpret and represent change in quantities, individually or in conjunction. Through the action buttons, I intended to foster students' making predictions prior to viewing animations and/or graphs.

When developing the task sequence, I drew on variation theory (Marton & Booth, 1997), introducing differences in the types of measurable attributes (e.g., distance from start, height from the ground, and width from the center) and/or the representation of the attributes on the horizontal and vertical axes (e.g., distance on horizontal axis, height on vertical axis and vice versa). For more details, see Johnson (2015b).

TASK DESIGN PRINCIPLES

Incorporate measurable attributes that are not time

Drawing on the work of Newton, Thompson (2012) distinguished between conceptual time (time as a measurable attribute that can be separated from an individual's experience of passing time) and experiential (passing) time. Because students may not conceive of time separately from their experience of passing time, I recommend that tasks designed to foster students' covariational reasoning incorporate attributes that are not time. This is not to say that secondary students working on tasks involving time do not engage in covariational reasoning (cf. Ellis et al., 2015). However, incorporating attributes other than time may provide opportunities for students first to envision variation within passing time, which then may allow them incorporate time into the variation, thereby fostering their ability to separate time from the changing attributes (cf. Thompson, 2012).

Problematize the determination of numerical amounts of change

Students demonstrating smooth conceptions of change envision change as continuing. Yet, interacting with DCEs representing attributes as continually changing is insufficient to foster students' smooth conceptions of change. In a PreCalculus course designed to foster university students' covariational reasoning, instructional tasks provide students opportunities to conceive of changing quantities prior to determining numerical amounts (Thompson & Carlson, in press). I recommend that tasks for secondary students follow this lead, by providing students opportunities to envision attributes as changing continually prior to incorporating numerical amounts.

Vary the representation of measurable attributes

Frequently, tasks incorporating a Cartesian coordinate system represent time on the horizontal axis. Consequently, students may conceive of other attributes represented on horizontal axes as varying within their experience of passing time (or as passing time themselves). Tasks involving the same attributes represented on different axes in a Cartesian coordinate system afforded prospective mathematics teachers opportunities to engage in covariational reasoning when interpreting graphs (Moore, Silverman, Paoletti, & LaForest, 2014). I recommend that task sequences for secondary students include tasks representing the same attributes on different axes to provide students opportunities to envision attributes as changing separately from their experience of passing time.

BRIEF RESULTS

During a series of six clinical interviews with one, two or three students, I implemented a Ferris wheel task sequence with a total of five ninth grade students (~15 yrs) enrolled in an introductory Algebra course. All five students demonstrated smooth conceptions of variation by the end of the interviews, and four of the students demonstrated covariational reasoning. Furthermore, two students demonstrated attention to variation in the intensity of change, as evidenced by sketching graphs with different concavities to account for variation in change occurring in a single direction. In addition, all students demonstrated shifts to more robust reasoning during the interview sequence, with shifts occurring when students were working with the dynamic segments or encountering attributes represented on different axes.

DISCUSSION/IMPLICATIONS

To introduce key concepts such as function, I recommend that instructional tasks not only foster secondary students' covariational reasoning, but also provide opportunities for students to develop and use smooth conceptions of variation. The technology-rich Ferris wheel tasks problematize numerical calculations while incorporating attributes (other than time) measurable with one-dimensional units. Such tasks hold promise not only for fostering students' covariational reasoning, but also for providing students opportunities to develop and use smooth conceptions of variation.

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